Intelligent Control of a Two-Link Flexible-Joint Robot, Using Backstepping, Neural Networks, and Direct Method

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***Abstract* – We present a state-feedback control of a two-link flexible-joint robot. First, we obtain desired control laws from Lyapunov's second method. Then, we use three-layer neural networks to learn unknown parts of the desired control laws. In this way, the control algorithm does not require the mathematical model representing the robot. We use smooth variable structure controller to handle the uncertainties from neural network approximation and external disturbances. To show the effectiveness and practicality of this control algorithm, we performed an experiment on one of the robots in our laboratory.**

Index Terms – Flexible-joint robot, Intelligent control, Backstepping, Variable structure control.

I. Introduction

The joint flexibility exists in most robots. It arises from driving components such as actuators, gear teeth, or transmission belts. In some applications, the designers incorporate flexible joints into their products intentionally to absorb impact force and to reduce damage to the parts from accidental collision.

Controller designers should explicitly include joint flexibility in their design because joint resonant frequencies, which are located within the control bandwidth, can be excited and cause severe oscillations. The experiment in [1] suggested that the designers should consider joint flexibility in both modeling and control design.

Controller design of two-link flexible-joint robot is challenging because its model is much more complicated than those of rigid-joint robot and one-link flexible-joint robot. Besides, the number of degree of freedom is twice the number of control inputs, which results in the lost of matching property between nonlinearities and the inputs and the lost of passivity from inputs to link velocities.

References [2] and [3] offer two choices of slow and fast variables in order to transform the dynamical model of the flexible-joint robot into the standard singular perturbation model. Slow-control input, which adds damping to the system, drives the closed-loop system to a quasi-steady state system that has the structure of a rigid-joint robot. Then, fast-control input can be designed using available techniques for the rigid-joint robot.

Reference [4] presents static feedback linearization method. Under the assumption that the kinetic energy of the motor is due mainly to its own rotation, the flexible-joint robot model is feedback linearizable. Reference [5] relaxes this assumption, and applies the so-called dynamic feedback linearization method to a more general robot model. Reference [6] offers a good comparison of three types of controllers: controller developed from decoupled model, backstepping controller, and passivity-based controller.

Newer results use intelligent systems to learn some or all of the unknown parts of the robot model. Reference [7] extends the work in [2] to the case where model uncertainties exist in the system. They use radial basis function networks to estimate unknown functions, and use discontinuous variable-structure controller to provide robustness to the closed-loop system. Reference [8] uses combinations of orthonormal basis functions to estimate unknown functions; they also present an experiment result of one-link flexible-joint robot. Reference [9] uses feedback linearization method and Takagi-Sugeno fuzzy system to replace model uncertainties.

In this paper, we consider the trajectory-tracking task of a two-link flexible-joint robot in horizontal plane. The controller algorithm does not require closed-form mathematical model of the robot. We design control laws from Lyapunov's second method using backstepping structure. Then, three-layer neural networks are used to estimate unknown parts of the desired control laws – usually called direct method by adaptive control community. We use variable structure controller to provide robustness to the system against uncertainties from the estimation errors, actuator nonlinearities, and external disturbances.

We organize this paper as follows. Section II contains details on the robot and the experiment setup. Section III contains three-layer neural network background and controller design. Section IV contains experiment results. Section V is conclusion of the paper.

II. A Two-Link Flexible-Joint Robot

Fig. 1 depicts a robot, for which we are designing the controller. The robot operates in horizontal plane, has two links and two motors. Input torque u_1 is applied to the first motor, which drives the first sprocket through a chain. The sprocket is attached to the first link via the first torsional spring that provides joint flexibility. The second motor is situated on the first link. Input torque u_2 is applied to the second motor which drives the second sprocket. The second sprocket is attached to the second link via the second torsional spring. Note that the second motor's shaft does not share the same axis with the axis of rotation of the second link. This setting is more practical than the shared-axis cases commonly treated in existing literature.

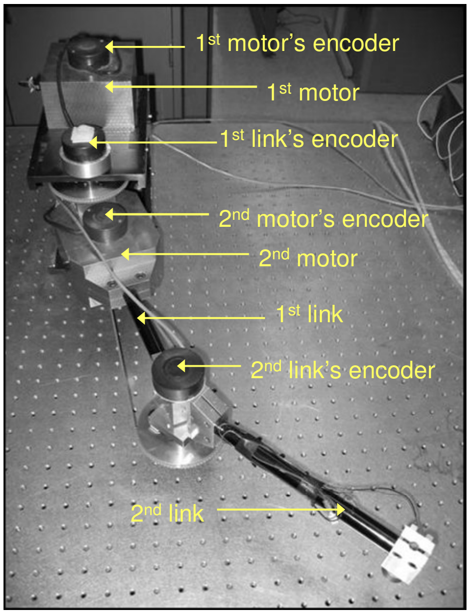

Fig. 1. Photograph of the two-link flexible-joint robot in our laboratory.

There are four optical encoders; each measures angular positions of the two links and the two motors. Angular velocities are obtained from numerical differentiation of the position signals. Two current amplifiers supply current to the two motors.

Fig. 2 depicts overall experiment setup. We use Labview 7.1, Labview Real-Time Module, and Labview FPGA Module to perform hardware-in-the-loop experiment. The data acquisition board is National Instruments' PCI-7831R.

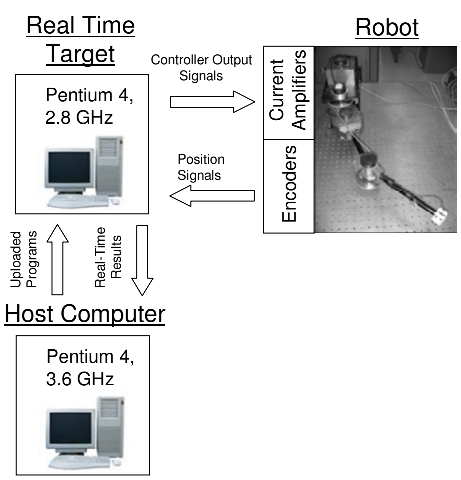

Fig. 2. Diagram showing overall experiment setup.

We let θ_1 be absolute angular position of the first link, θ_2 be relative angular position of the second link, θ_3 be absolute angular position of the first motor, and θ_4 be relative angular position of the second motor. If we let $x_1 = [x_{11}, x_{12}]^T = [\theta_1, \theta_2]^T$ and $x_2 = [x_{21}, x_{22}] = [\dot{\theta}_1, \dot{\theta}_2]^T$ be state vectors representing link position and velocity, $x_3 = [x_{31}, x_{32}] = [\theta_3, \theta_4]^T$ and $x_4 = [x_{41}, x_{42}] = [\dot{\theta}_3, \dot{\theta}_4]^T$ be state vectors representing motor position and velocity, and $u = [u_1, u_2]^T$ be input vector representing input torque, reference [10] shows that the equations of motion of this robot can be put in the following state space form:

$$\begin{aligned}
\dot{x}_1 &= x_2 + d_{a1}(\bar{x}_4),\\
\dot{x}_2 &= f_2(\bar{x}_2) + g_2(\bar{x}_2)(x_3 + d_{a2}(\bar{x}_4)),\\
\dot{x}_3 &= x_4 + d_{a3}(\bar{x}_4),\\
\dot{x}_4 &= f_4(\bar{x}_4) + g_4(\bar{x}_4)(u + d_{a4}(\bar{x}_4)),\\
y &= x_1,
\end{aligned} \tag{1}$$

where $\bar{x}_i = \{x_{11}, x_{12}, \ldots, x_{i1}, x_{i2}\}$; $f_2, f_4 \in \mathbb{R}^2$ and $g_2, g_4 \in \mathbb{R}^{2\times 2}$ are vectors and matrices that contain smooth functions; $d_{ai}(\bar{x}_4) = [d_{ai1}, d_{ai2}]^T$ is additive disturbance vector that may depend on all states.

In the next section, we will design a controller for the system in the form of (1), where f_2, f_4, g_2, and g_4 are unknown. The disturbance d_{ai} is bounded by an unknown constant.

III. Controller Design

We, first, present some basics of the three-layer neural network followed by the controller design.

A. *Three-Layer Neural Network*

Fig. 3 depicts a three-layer neural network. Suppose, a scalar-valued continuous function $h(z_1, z_2, \ldots, z_n): \mathbb{R}^n \to \mathbb{R}$ is to be estimated by the neural network. We have $z_1, z_2, \ldots, z_n, 1$ as inputs to the neural network. Variables in the network are defined as follows:

$$\begin{aligned}
&\bar{Z} = [z_1, z_2, \ldots, z_n, 1]^T \in \mathbb{R}^{n+1},\\
&V = [v_1, v_2, \ldots, v_l] \in \mathbb{R}^{(n+1)\times l},\\
&v_i = [v_{i1}, v_{i2}, \ldots, v_{i(n+1)}]^T \in \mathbb{R}^{n+1}, i = 1, 2, \ldots, l,\\
&S(V^T\bar{Z}) = [s(v_1^T\bar{Z}), s(v_2^T\bar{Z}), \ldots, s(v_l^T\bar{Z}), 1]^T \in \mathbb{R}^{l+1},\\
&W = [w_1, w_2, \ldots, w_l, w_{l+1}]^T \in \mathbb{R}^{l+1},\\
&h(W, V, z_1, z_2, \ldots, z_n) = W^T S(V^T\bar{Z}) \in \mathbb{R}.
\end{aligned}$$

$s(\bullet)$ is a sigmoid function

$$s(x) = 1/(1+e^{-x}), \forall x \in \mathbb{R}.$$

This network can uniformly approximate any scalar-valued continuous function to any arbitrary accuracy with some constant ideal weights W^*, V^*, and some appropriate number of hidden-layer nodes, l^*, as was proved in [11]. From the universal approximation property, we have

$$h(z_1, z_2, \ldots, z_n) = W^{*T} S(V^{*T}\bar{Z}) + \varepsilon, \tag{2}$$

where $\|\varepsilon\| < \varepsilon_U$ is approximation error with unknown $\varepsilon_U > 0$

providing that $h(\cdot)$ is defined on a compact set Ω_z.

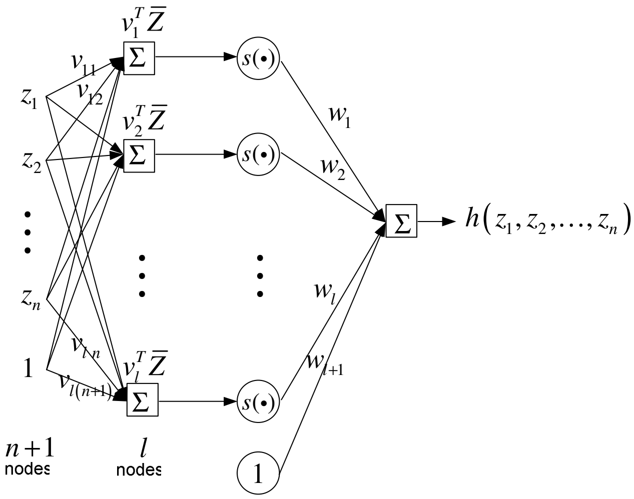

Fig. 3. A three-layer neural network. The square represents node whose output contains adjustable parameters.

The ideal weights generally are unknown. However, in system identification application, the ideal weights are typically assumed constant and bounded as in the following Assumption.

Assumption 1: On the compact set Ω_z, the ideal neural network weights W^*, V^* are constant and bounded by $\|W^*\| \le W_U$, $\|V^*\|_F \le V_U$, $i = 1, \ldots, m$, where W_U and V_U are unknown.

Since ideal weights are unknown, let $\hat{W}$ and $\hat{V}$ be the estimates of W^* and V^* respectively. The estimate of the function h is given by

$$\hat{h}(z_1, z_2, \ldots, z_n) = \hat{W}^T S\left(\hat{V}^T \bar{Z}\right). \tag{3}$$

By using Lemma 3.6 in [12], the difference between neural network output with ideal and estimated weights are given by

$$\begin{aligned} \hat{W}^T S(\hat{V}^T \bar{Z}) - W^{*T} S(V^{*T} \bar{Z}) &= \tilde{W}^T (\hat{S} - \hat{S}' \hat{V}^T \bar{Z}) \\ &+ \hat{W}^T \hat{S}' \tilde{V}^T \bar{Z} + d_u, \end{aligned} \tag{4}$$

where

$$\tilde{W} = \hat{W} - W^*, \tilde{V} = \hat{V} - V^*, \hat{S} = S(\hat{V}^T \bar{Z}) \in \mathbb{R}^{l+1},$$
$$\hat{S}' = diag\left\{\hat{s}_1', \hat{s}_2', \ldots, \hat{s}_l', 0\right\} \in \mathbb{R}^{(l+1)\times(l+1)},$$
$$\hat{s}_i' = s'(\hat{v}_i^T \bar{Z}) = d[s(z_a)]/dz_a\big|_{z_a = \hat{v}_i^T \bar{z}} \in \mathbb{R}, i = 1, 2, \ldots, l,$$
$$s(x) = 1/(1+e^{-x}), \forall x \in \mathbb{R}.$$

The residual term d_u is bounded by

$$|d_u| \le \|V^*\|_F \left\|\bar{Z} \hat{W}^T \hat{S}'\right\|_F + \|W^*\| \left\|\hat{S}' \hat{V}^T \bar{Z}\right\| + |W^*|_1. \tag{5}$$

Note that $\tilde{W}$ and $\tilde{V}$ appear linearly in (4). This is important since from Assumption 1, $\dot{\tilde{W}} = \dot{\hat{W}}$ and $\dot{\tilde{V}} = \dot{\hat{V}}$, therefore the weight adaptation laws can be easily designed using this linear structure.

B. Backstepping and Variable Structure Controller

In this section, we design a controller that makes link angular positions θ_1 and θ_2 track desired values whereas all closed-loop signals remain bounded. We require the following assumptions.

Assumption 2: The additive disturbance $d_{aik}(\bar{x}_4)$, where $i = 1, \ldots, 4$, $k = 1, 2$, is bounded by $\|d_{aik}(\bar{x}_4)\| < d_{aikU}$, where d_{aikU} is unknown constant.

Assumption 3: The inverses of $g_i, \forall i = 2, 4$, matrices in (1) are positive definite.

Assumption 4: The desired trajectory $x_{1d} = [x_{11d}, x_{12d}]^T$ is sufficiently smooth.

In backstepping design, we try to reduce the error between actual state and desired state of each subsystem. The tracking error is the error of the first subsystem. Let $e_i = [e_{i1}, e_{i2}]^T = x_i - x_{id}$, $i = 1, \ldots, 4$ be those errors.

Step 1:

Let the virtual control law of the first subsystem be

$$x_{2d} = -c_1 e_1 + \dot{x}_{1d} + u_{2dvsc} = [x_{21d}, x_{22d}]^T,$$

where c_1 is a positive design parameter, u_{2dvsc} is variable structure control law to be designed. From Assumption 2, we have the following inequality

$$\|d_{a1j}(\bar{x}_4)\| \le K_{1j}^{*T} \phi_{1j}, \forall j = 1, 2,$$

where $K_{1j}^* = d_{a1jU}$, $\phi_{1j} = 1$.

We let the smooth variable structure control law be in the form $u_{2dvsc} = [u_{2dvsc1}, u_{2dvsc2}]^T$, where

$$u_{2dvscj} = -\hat{K}_{1j} \bar{\phi}_{1j} = -\hat{K}_{1j}\left(\frac{2}{\pi}\arctan\left(\frac{e_{1j}}{\mu_{1j}}\right)\right),$$

μ_{1j} is a small positive design parameter, $\hat{K}_{1j}$ approximates K_{1j}^* with error given by $\tilde{K}_{1j} = \hat{K}_{1j} - K_{1j}^*$.

The time derivative of the error of the first subsystem becomes

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_{1d} = (x_2 + d_{a1}) - \dot{x}_{1d} = (e_2 + x_{2d} + d_{a1}) - \dot{x}_{1d} \\ &= e_2 - c_1 e_1 + u_{2dvsc} + d_{a1}. \end{aligned}$$

Let the weight update law be

$$\dot{\hat{K}}_{1j} = \Gamma_{k1j}[\bar{\phi}_{1j} e_{1j} - \sigma_{k1j} \hat{K}_{1j}],$$

where $\Gamma_{k1j} > 0, \sigma_{k1j} > 0, \forall j = 1, 2$.

Using the following facts

$$\begin{aligned} &2\tilde{K}^T \hat{K} = \|\tilde{K}\|^2 + \|\hat{K}\|^2 - \|K^*\|^2 \ge \|\tilde{K}\|^2 - \|K^*\|^2, \\ &0 \le |\alpha| - \alpha \frac{2}{\pi}\arctan\left(\frac{\alpha}{\mu}\right) \le 0.2785\mu, \forall \alpha \in \mathbb{R}, \end{aligned} \tag{6}$$

the time derivative of the Lyapunov function

$$V_1 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} \sum_{j=1}^{2} \tilde{K}_{1j}^T \Gamma_{k1j}^{-1} \tilde{K}_{1j},$$

is given by

$$\begin{aligned} \dot{V}_1 &= e_1^T \dot{e}_1 + \sum_{j=1}^{2} \tilde{K}_{1j}^T \Gamma_{k1j}^{-1} \dot{\hat{K}}_{1j} \\ &\le e_1^T e_2 - e_1^T c_1 e_1 + \sum_{j=1}^{2} \left(-K_{1j}^{*T} \bar{\phi}_{1j} e_{1j} + K_{1j}^{*T} \phi_{1j} \left| e_{1j} \right| - \tilde{K}_{1j}^T \sigma_{k1j} \hat{K}_{1j} \right) \\ &\le e_1^T e_2 - e_1^T c_1 e_1 - \sum_{j=1}^{2} \sigma_{k1j} \left\| \tilde{K}_{1j} \right\|^2 / 2 + \xi_1, \end{aligned}$$

where

$$\xi_1 = \sum_{j=1}^{2} \left(0.2785 \mu_{1j} d_{a1jU} + \sigma_{k1j} \left\| K_{1j}^* \right\|^2 / 2 \right).$$

Step 2:

This step is different from the first step because there are unknown functions f_2 and g_2 in this second subsystem.

The time-derivative of the error of the second subsystem is given by

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = f_2 + g_2 \left(x_3 + d_{a2} \right) - \dot{x}_{2d}.$$

Suppose we know f_2 and g_2, assume there is no disturbance d_{a1} and d_{a2} for now, we can choose the virtual control input as

$$x_{3d}^* = -e_1 - c_2 e_2 - g_2^{-1} \left(f_2 - \dot{x}_{2d} \right). \tag{7}$$

Using the Lyapunov function $V_2 = e_1^T e_1 / 2 + e_2^T g_2^{-1} e_2 / 2,$ we have $\dot{V}_2 = -c_1 e_1^T e_1 - c_2 e_2^T e_2$ which is negative definite, therefore, the error e_1 and e_2 converge to zero.

Since we do not know f_2 and g_2, we need to modify the ideal control input x_{3d}^*. From (7), the unknown part is $h_2^*(Z_2) \triangleq g_2 \left(\bar{x}_2 \right)^{-1} \left[f_2(\bar{x}_2) - \dot{x}_{2d} \right] = \left[h_{21}^*, h_{22}^* \right]^T \in \mathbb{R}^2$, where h_{2j}^* is a scalar-valued continuous function of x_1, x_2, and $\dot{x}_{2jd}$. We proceed by estimating each unknown part using a three-layer neural network. From (2), we have

$$x_{3d}^* = -e_1 - c_2 e_2 - \begin{bmatrix} W_{21}^{*T} S_{21} (V_{21}^{*T} \bar{Z}_{21}) - \varepsilon_{21} \\ W_{22}^{*T} S_{22} (V_{22}^{*T} \bar{Z}_{22}) - \varepsilon_{22} \end{bmatrix}.$$

W_{2j}^* and V_{2j}^* are unknown. Let $\hat{W}_{2j}$ and $\hat{V}_{2j}$ be their estimates and add smooth variable structure control law to handle the uncertainties, we have the virtual control law

$$x_{3d} = -e_1 - c_2 e_2 - \begin{bmatrix} \hat{W}_{21}^T S_{21} (\hat{V}_{21}^T \bar{Z}_{21}) \\ \hat{W}_{22}^T S_{22} (\hat{V}_{22}^T \bar{Z}_{22}) \end{bmatrix} + u_{3dvsc} = \left[x_{31d}, x_{32d} \right]^T.$$

From (2), (5), and Assumption 2, we have

$$\left| d_{u2j} \right| + \left| \varepsilon_{2j} \right| + \left| d_{a2j} \right| \le K_{2j}^{*T} \phi_{2j},$$

where

$$K_{2j}^* = \left[\left\| V_{2j}^* \right\|_F, \ \left\| W_{2j}^* \right\|, \ \left\| W_{2j}^* \right\|_1 + \varepsilon_{2jU} + d_{a2jU} \right]^T,$$

$$\phi_{2j} = \left[\left\| \bar{Z}_{2j} \hat{W}_{2j}^T \hat{S}_{2j}' \right\|_F, \ \left\| \hat{S}_{2j}' \hat{V}_{2j}^T \bar{Z}_{2j} \right\|, \ 1 \right]^T, \forall j = 1, 2.$$

We let the smooth variable structure control law be $u_{3dvsc} = \left[u_{3dvsc1}, u_{3dvsc2} \right]^T$, where

$$u_{3dvscj} = -\hat{K}_{2j}^T \bar{\phi}_{2j},$$

$$\bar{\phi}_{2j} = \begin{bmatrix} \left\| \bar{Z}_{2j} \hat{W}_{2j}^T \hat{S}_{2j}' \right\|_F \frac{2}{\pi} \arctan \left(\frac{e_{2j}}{\mu_{2j}} \left\| \bar{Z}_{2j} \hat{W}_{2j}^T \hat{S}_{2j}' \right\|_F \right) \\ \left\| \hat{S}_{2j}' \hat{V}_{2j}^T \bar{Z}_{2j} \right\| \frac{2}{\pi} \arctan \left(\frac{e_{2j}}{\mu_{2j}} \left\| \hat{S}_{2j}' \hat{V}_{2j}^T \bar{Z}_{2j} \right\| \right) \\ \frac{2}{\pi} \arctan \left(\frac{e_{2j}}{\mu_{2j}} \right) \end{bmatrix}.$$

The time derivative of the error of the second subsystem becomes

$$\dot{e}_2 = g_2 \left\{ \begin{array}{l} e_3 - e_1 - c_2 e_2 \\ + \begin{bmatrix} \varepsilon_{21} - \tilde{W}_{21}^T (\hat{S}_{21} - \hat{S}_{21}' \hat{V}_{21}^T \bar{Z}_{21}) \\ - \hat{W}_{21}^T \hat{S}_{21}' \tilde{V}_{21}^T \bar{Z}_{21} - d_{u21} - \hat{K}_{21}^T \bar{\varphi}_{21} + d_{a21} \\ \hdashline \varepsilon_{22} - \tilde{W}_{22}^T (\hat{S}_{22} - \hat{S}_{22}' \hat{V}_{22}^T \bar{Z}_{22}) \\ - \hat{W}_{22}^T \hat{S}_{22}' \tilde{V}_{22}^T \bar{Z}_{22} - d_{u22} - \hat{K}_{22}^T \bar{\varphi}_{22} + d_{a22} \end{bmatrix} \end{array} \right\}.$$

Let the weight update laws be

$$\begin{aligned} \dot{\hat{W}}_{2j} &= \Gamma_{w2j} [(\hat{S}_{2j} - \hat{S}_{2j}' \hat{V}_{2j}^T \bar{Z}_{2j}) e_{2j} - \sigma_{w2j} \hat{W}_{2j}], \\ \dot{\hat{V}}_{2j} &= \Gamma_{v2j} [\bar{Z}_{2j} \hat{W}_{2j}^T \hat{S}_{2j}' e_{2j} - \sigma_{v2j} \hat{V}_{2j}], \\ \dot{\hat{K}}_{2j} &= \Gamma_{k2j} [\bar{\phi}_{2j} e_{2j} - \sigma_{k2j} \hat{K}_{2j}], \end{aligned}$$

and using the facts (6), and

$$2 \tilde{W}^T \hat{W} = \left\| \tilde{W} \right\|^2 + \left\| \hat{W} \right\|^2 - \left\| W^* \right\|^2 \ge \left\| \tilde{W} \right\|^2 - \left\| W^* \right\|^2,$$

$$2 tr \left\{ \tilde{V}^T \hat{V} \right\} = \left\| \tilde{V} \right\|_F^2 + \left\| \hat{V} \right\|_F^2 - \left\| V^* \right\|_F^2 \ge \left\| \tilde{V} \right\|_F^2 - \left\| V^* \right\|_F^2,$$

the time derivative of the Lyapunov function

$$\begin{aligned} V_2 = V_1 + \frac{1}{2} e_2^T g_2^{-1} e_2 + \frac{1}{2} \sum_{j=1}^{2} \Big(& \tilde{W}_{2j}^T \Gamma_{w2j}^{-1} \tilde{W}_{2j} \\ & + tr \left\{ \tilde{V}_{2j}^T \Gamma_{v2j}^{-1} \tilde{V}_{2j} \right\} + \tilde{K}_{2j}^T \Gamma_{k2j}^{-1} \tilde{K}_{2j} \Big), \end{aligned}$$

derived similar to what in step 1, is given by

$$\begin{aligned} \dot{V}_2 \le e_2^T e_3 - \sum_{j=1}^{2} & \left(\sigma_{w2j} \left\| \tilde{W}_{2j} \right\|^2 / 2 + \sigma_{v2j} \left\| \tilde{V}_{2j} \right\|_F^2 / 2 \right) \\ & - \sum_{i=1}^{2} \left(e_i^T c_i e_i - \xi_i \right) - \sum_{i=1}^{2} \sum_{j=1}^{2} \left(\sigma_{kij} \left\| \tilde{K}_{ij} \right\|^2 / 2 \right), \end{aligned}$$

where

$$\xi_2 = \sum_{j=1}^{2}\Big[0.2785\mu_{2j}\left(\left\|V_{2j}^*\right\|_F + \left\|W_{2j}^*\right\| + \left\|W_{2j}^*\right\|_1 + \varepsilon_{2jU} + d_{a2jU}\right)$$
$$+\sigma_{w2j}\left\|W_{2j}^*\right\|^2/2 + \sigma_{v2j}\left\|V_{2j}^*\right\|_F^2/2 + \sigma_{k2j}\left\|K_{2j}^*\right\|^2/2\Big].$$

Step 3:

This step is similar to step 1. Let the virtual control law be

$$x_{4d} = -e_2 - c_3 e_3 + \dot{x}_{3d} + u_{4dvsc} = \left[x_{41d}, x_{42d}\right]^T.$$

We use the smooth variable structure control law as $u_{4dvsc} = \left[u_{4dvsc1}, u_{4dvsc2}\right]^T$, where

$$u_{4dvscj} = -\hat{K}_{3j}\bar{\phi}_{3j} = -\hat{K}_{3j}\left(\frac{2}{\pi}\arctan\left(\frac{e_{3j}}{\mu_{3j}}\right)\right).$$

The weight update law is $\dot{\hat{K}}_{3j} = \Gamma_{k3j}[\bar{\phi}_{3j}e_{3j} - \sigma_{k3j}\hat{K}_{3j}]$.

Using similar derivation to what in step 1, the time derivative of the Lyapunov function

$$V_3 = V_1 + V_2 + \frac{1}{2}e_3^T e_3 + \frac{1}{2}\sum_{j=1}^{2}\tilde{K}_{3j}^T\Gamma_{k3j}^{-1}\tilde{K}_{3j},$$

is given by

$$\dot{V}_3 \le e_3^T e_4 - \sum_{j=1}^{2}\left(\sigma_{w2j}\left\|\tilde{W}_{2j}\right\|^2/2 + \sigma_{v2j}\left\|\tilde{V}_{2j}\right\|_F^2/2\right)$$
$$-\sum_{i=1}^{3}\left(e_i^T c_i e_i - \xi_i\right) - \sum_{i=1}^{3}\sum_{j=1}^{2}\left(\sigma_{kij}\left\|\tilde{K}_{ij}\right\|^2/2\right),$$

where $\xi_3 = \sum_{j=1}^{2}\left(0.2785\mu_{3j}d_{a3jU} + \sigma_{k3j}\left\|K_{3j}^*\right\|^2/2\right).$

Step 4:

This is the last step and is similar to what in step 2. Let the actual control law be

$$u = -e_3 - c_4 e_4 - \begin{bmatrix}\hat{W}_{41}^T S_{41}(\hat{V}_{41}^T\bar{Z}_{41}) \\ \hat{W}_{42}^T S_{42}(\hat{V}_{42}^T\bar{Z}_{42})\end{bmatrix} + u_{5dvsc} = \left[u_1, u_2\right]^T.$$

The smooth variable structure control law is $u_{5dvsc} = \left[u_{5dvsc1}, u_{5dvsc2}\right]^T$, where

$$u_{5dvscj} = -\hat{K}_{4j}{}^T\bar{\phi}_{4j},$$

$$\bar{\phi}_{4j} = \begin{bmatrix}\left\|\bar{Z}_{4j}\hat{W}_{4j}^T\hat{S}_{4j}'\right\|_F \dfrac{2}{\pi}\arctan\left(\dfrac{e_{4j}}{\mu_{4j}}\left\|\bar{Z}_{4j}\hat{W}_{4j}^T\hat{S}_{4j}'\right\|_F\right) \\ \left\|\hat{S}_{4j}'\hat{V}_{4j}^T\bar{Z}_{4j}\right\|\dfrac{2}{\pi}\arctan\left(\dfrac{e_{4j}}{\mu_{4j}}\left\|\hat{S}_{4j}'\hat{V}_{4j}^T\bar{Z}_{4j}\right\|\right) \\ \dfrac{2}{\pi}\arctan\left(\dfrac{e_{4j}}{\mu_{4j}}\right)\end{bmatrix}.$$

Let the weight update laws be

$$\dot{\hat{W}}_{4j} = \Gamma_{w4j}[(\hat{S}_{4j} - \hat{S}_{4j}'\hat{V}_{4j}{}^T\bar{Z}_{4j})e_{4j} - \sigma_{w4j}\hat{W}_{4j}],$$

$$\dot{\hat{V}}_{4j} = \Gamma_{v4j}[\bar{Z}_{4j}\hat{W}_{4j}{}^T\hat{S}_{4j}'e_{4j} - \sigma_{v4j}\hat{V}_{4j}],$$
$$\dot{\hat{K}}_{4j} = \Gamma_{k4j}[\bar{\phi}_{4j}e_{4j} - \sigma_{k4j}\hat{K}_{4j}].$$

The time derivative of the Lyapunov function

$$V_4 = V_1 + V_2 + V_3 + \frac{1}{2}e_4^T g_4^{-1}e_4 + \frac{1}{2}\sum_{j=1}^{2}\Big(\tilde{W}_{4j}^T\Gamma_{w4j}^{-1}\tilde{W}_{4j}$$
$$+tr\left\{\tilde{V}_{4j}^T\Gamma_{v4j}^{-1}\tilde{V}_{4j}\right\} + \tilde{K}_{4j}^T\Gamma_{k4j}^{-1}\tilde{K}_{4j}\Big),$$

derived similar to what in step 2, is given by

$$\dot{V}_4 \le -\sum_{j=1}^{2}\left(\sigma_{w2j}\left\|\tilde{W}_{2j}\right\|^2/2 + \sigma_{v2j}\left\|\tilde{V}_{2j}\right\|_F^2/2\right)$$
$$-\sum_{j=1}^{2}\left(\sigma_{w4j}\left\|\tilde{W}_{4j}\right\|^2/2 + \sigma_{v4j}\left\|\tilde{V}_{4j}\right\|_F^2/2\right)$$
$$-\sum_{i=1}^{4}\left(e_i^T c_i e_i - \xi_i\right) - \sum_{i=1}^{4}\sum_{j=1}^{2}\left(\sigma_{kij}\left\|\tilde{K}_{ij}\right\|^2/2\right),$$

where

$$\xi_4 = \sum_{j=1}^{2}\Big[0.2785\mu_{4j}\left(\left\|V_{4j}^*\right\|_F + \left\|W_{4j}^*\right\| + \left\|W_{4j}^*\right\|_1 + \varepsilon_{4jU} + d_{a4jU}\right)$$
$$+\sigma_{w4j}\left\|W_{4j}^*\right\|^2/2 + \sigma_{v4j}\left\|V_{4j}^*\right\|_F^2/2 + \sigma_{k4j}\left\|K_{4j}^*\right\|^2/2\Big].$$

Let

$$\varsigma = \min\left\{\min_{i=1,3}\left\{\frac{c_i}{0.5}\right\}, \min_{j=2,4}\left\{\frac{c_j}{0.5\left\|g_j^{-1}\right\|}\right\}\right\} > 0,\ \delta = \sum_{k=1}^{4}\xi_k \ge 0$$

and choose

$$\sigma_{wlj} \ge \varsigma\lambda_{\max}\left\{\Gamma_{wlj}^{-1}\right\}, \sigma_{vlj} \ge \varsigma\lambda_{\max}\left\{\Gamma_{vlj}^{-1}\right\},$$
$$\sigma_{kij} \ge \varsigma\lambda_{\max}\left\{\Gamma_{kij}^{-1}\right\};\ l = 2,4;\ i = 1,\ldots,4;\ j = 1,2,$$

we have $\dot{V}_4 \le -\varsigma V_4 + \delta$.

From this point on, you can use standard nonlinear analysis technique given, for example, in [13] to conclude that all error trajectories are globally uniformly ultimately bounded. The ultimate bound, time the trajectories enter the bound, and all-time exponential-decay upper bound can also be found, but are omitted here.

IV. Experiment Results

There are 4 neural networks; each has 3 hidden nodes. Inputs to each neural network are

$$\bar{Z}_{21} = \left\{x_{11}, x_{12}, x_{21}, x_{22}, \dot{x}_{21d}, 1\right\},$$
$$\bar{Z}_{22} = \left\{x_{11}, x_{12}, x_{21}, x_{22}, \dot{x}_{22d}, 1\right\},$$
$$\bar{Z}_{41} = \left\{x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, \dot{x}_{41d}, 1\right\},$$
$$\bar{Z}_{42} = \left\{x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, \dot{x}_{42d}, 1\right\}.$$

Neural network and controller design parameters are as follows:

$$\Gamma_{wij} = \Gamma_{vij} = \Gamma_{kij} = 0.0001, c_i = 5,$$
$$\sigma_{wij} = \sigma_{vij} = \sigma_{kij} = 0.1, \mu = 1.$$

All initial values are set to zeros. Sampling period is 10 ms. The desired trajectory is obtained from passing square wave signal of amplitude 3, and 40-second period into the filter $1/(s+2)^3$.

Experiment result is given in Fig. 4. The control system achieves good overall tracking performance as can be seen from the results in part (a) and (b). Both link angular positions θ_1 and θ_2 are able to follow their desired trajectories quite closely. Part (c) to (f) show estimated values of the unknown functions. Part (g) and (h) are control inputs to the two current amplifiers. Their values can be converted to voltage by multiplying with $10/2^{16}/2$.

V. CONCLUSION

The controller achieves good tracking performance. However, there are some interesting questions left, probably, as future work. First, the controller is designed based on the assumption that the actual robot is in the nonlinear form (1). The fact that the actual robot may not be exactly in this form may degrade the controller performance. Second, how will the control system handle time-varying case, for example, the change in payload?

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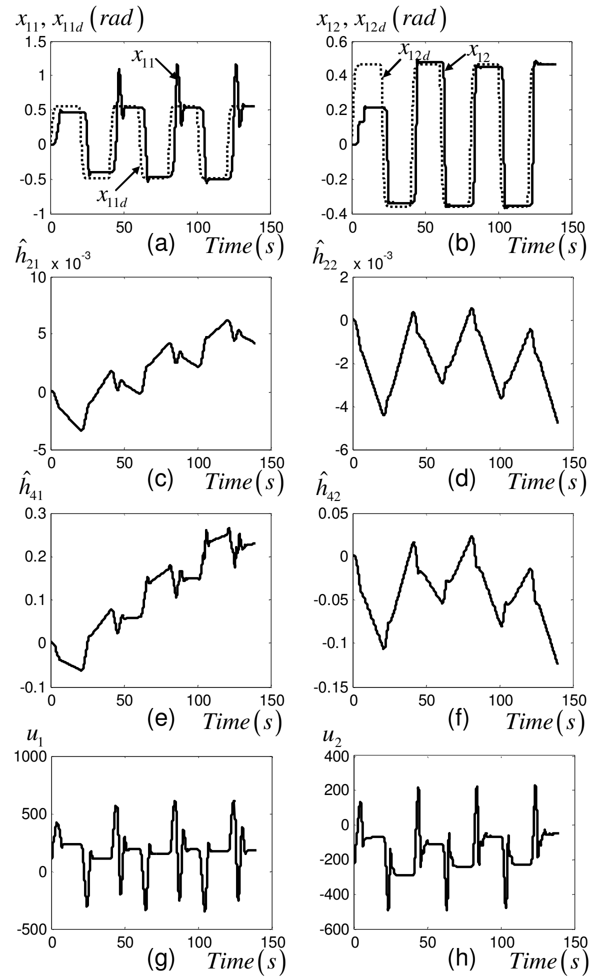

Fig. 4. Experiment result in 140 seconds. (a) θ_1 versus its desired trajectory θ_{1d}. (b) θ_2 versus its desired trajectory θ_{2d}. (c) Estimated function $\hat{h}_{21}$. (d) Estimated function $\hat{h}_{22}$. (e) Estimated function $\hat{h}_{41}$. (f) Estimated function $\hat{h}_{42}$. (g) Control input u_1. (h) Control input u_2.