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Robust Input Shaping Using Backstepping Model Matching Control

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Abstract— **Input shaping performance deteriorates when the knowledge of the mode parameters is not accurate. Several robust input shapers were proposed at the expense of longer move time. A novel input shaping system, using backstepping and variable structure controllers to match the closed-loop system with a known reference model, is presented. The simple ZV input shaper can then be designed from the mode parameters of the reference model. Advantages over the existing robust input shapers include substantially larger amount of uncertainty in the mode parameters can be tolerated, substantially shorter move time that does not increase with insensitivity, application to nonlinear systems, and disturbance-induced vibration suppression.**

I. INTRODUCTION

Input shaping suppresses the residual vibration, occurring from moving a flexible system rapidly from point to point, by destructive interference of impulse responses. Given appropriate impulse amplitudes and applied times, an impulse response of the flexible system can be cancelled by another impulse response, resulting in zero residual vibration. This idea was originally proposed under the name Posicast control [1]. Ref. [2] extended the Posicast control to increase its robustness to parameter uncertainty, creating the so-called ZVD^k input shaper.

Because the original Posicast control is susceptible to parameter uncertainty, several researchers have proposed modifications to increase its robustness. In the ZVDk input shaper, higher-order derivatives of the zero residual vibration expression, taken with respect to the mode parameters, are included as additional constraints to solve for the input shaper. Ref. [3] proposed a so-called extra-insensitive (EI) shaper. In designing the EI shaper, a small amount of residual vibration, instead of zero, is allowed when the system model is exactly known. As a result, the robustness of the input shaper is increased. Ref. [4] applied the frequency sampling method to enforce constraints on the vibration amplitude over a specified range of natural frequencies. This input shaper is called specified-insensitivity (SI) shaper because the insensitivity to frequency variation can be specified by the designer. In [5], multiplication of a series of ZV input shapers in the Laplace domain was proposed. The impulse times of each ZV input shaper in the series are slightly perturbed, resulting in a so-called perturbation-based extra-insensitive input shapers (PEI-ISs). PEI-ISs were shown to be more robust to parameter variation than the EI shaper and to have less transient vibration than the EI and SI shapers. Ref. [6] proposed a so-called minimax input shaper. The optimization problem that creates this shaper aims to move the system from rest to rest while minimizing the maximum magnitude of the residual states over a range of uncertain parameter values. In [7], an input shaper was proposed to deal with the statistical nature of plant parameter variations. The knowledge of the probability distribution of the system natural frequency about its modeled value is taken into account when the shaper is designed. Ref. [8] presents an improvement on that of [7]. Using the approximate of the stochastic system state by finite-dimensional series expansion in the stochastic space reduces the computational expense of the shaper in [7] when the dimension of the parameters grows. In [9], a modified command filtering technique in discrete time was proposed to ensure a more uniform output for each discrete-time sample while the system parameters vary with time. In [10], an optimization problem having linear matrix inequalities (LMI) constraints was formulated to obtain a robust input shaper. Ref. [11] compared among several types of robust input shapers.

The previously proposed robust input shapers have more robustness to the mode parameter uncertainty at the price of having more impulses in the input shaper sequence. As a result, the shaped reference input takes longer time to reach its final value, resulting in longer move time.

In this paper, a system consisting of the backstepping and variable structure controllers [12] is proposed to match the closed-loop system having uncertain plant to a known reference model. The backstepping structure provides a means for the control effort to reach the unmatched uncertainty in the system whereas the variable structure controller handles the uncertainty. The zero-vibration (ZV) input shaper is placed outside of the closed-loop. It is designed from mode parameters of the reference model. Because the reference model is known exactly, the input shaper needs not be robust and can have short duration.

The proposed system in this paper has several advantages over the previously proposed robust input shapers and model-matching input shapers as follows:

- Substantially larger amount of uncertainty in the mode parameters can be tolerated. Simulation and experimental results with a flexible-joint robot manipulator have shown that the proposed backstepping model matching with input shaping

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(BMM-IS) technique has almost zero sensitivity to uncertainty in both natural frequency and damping ratio.

- Shaped reference input has short duration, and its duration does not increase with the amount of insensitivity, as in the case of the robust input shaping. This fact was illustrated via simulation with the flexible-joint robot manipulator.
- The system applies to nonlinear plant that can be put in a general strict-feedback form. The feedback control handles the nonlinearity. Simulation results have shown that percentage vibration of traditional robust input shapers increases with hard nonlinearity in the plant. However, the proposed technique achieves zero percentage vibration in the presence of hard nonlinearity.
- Uncertainties in the plant model as well as disturbances can appear in any part of the model. The matching assumption between the control effort and the uncertainty is not required because of the backstepping structure. From simulation, percentage vibration induced by a plant-input disturbance is lower with the proposed technique.

In this paper, Section 2 presents the proposed backstepping model matching with input shaping technique. Section 3 contains details on the flexible-joint robot manipulator that was used in simulation and experiment. Section 4 discusses the simulation and experimental results. Conclusions are given in Section 5.

II. Input Shaping Using Backstepping Model Matching

Fig. 1 shows a diagram of the proposed system. The dashed box contains the closed-loop system. By using the states x, the backstepping variable structure control input u_c is aimed to reduce the error e between the plant output y and the reference model output y_m. As a result, the closed-loop system, which is a mapping from the shaped reference input r_s to y will be close to the known reference model, which is a mapping from r_s to y_m. The input shaper then can be designed from mode parameters of the reference model, which are exactly known. Note that the input u to the plant is the summation of u_c and r_s. Because the input shaper does not need to be robust, the ZV input shaper, which has the shortest length and the least robustness, is used outside the loop. The baseline reference r_b is normally a step function.

The flexible plant is assumed to be a single-input-single-output (SISO) system in the strict-feedback form [13] with additive disturbances, that is,

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)(x_{i+1} + d_{ai}(\bar{x}_m)), \quad 1 \le i \le m-1, \\ \dot{x}_m &= f_m(\bar{x}_m) + g_m(\bar{x}_m)(u + d_{am}(\bar{x}_m)), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}$, are the state variables; $\bar{x}_i = \{x_1, \ldots, x_i\}$, $i = 1, \ldots, m$, are the sets of state variables; $u \in \mathbb{R}$ is the input to the plant; $y \in \mathbb{R}$ is the plant output; $f_i(\bullet), g_i(\bullet), i = 1, \ldots, m$ are the known smooth functions; $d_{ai}(\bar{x}_m), i = 1, \ldots, m$ are the unknown but bounded additive disturbances or uncertainties in the plant model with unknown bounds.

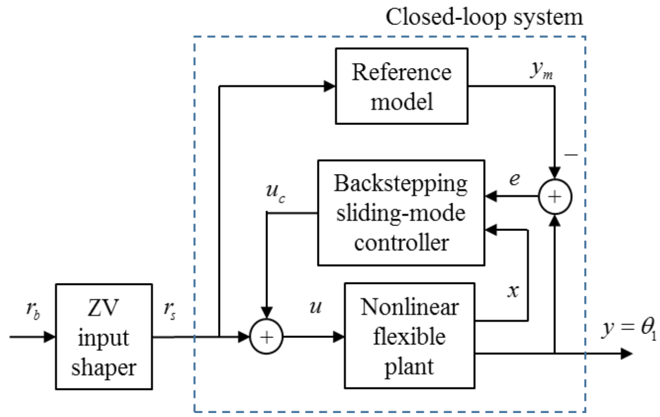

Fig. 1 Proposed backstepping model-matching control with input shaper.

For controller design and stability proof, the following assumptions are needed.

Assumption 1: Additive disturbances $d_{ai}(\bar{x}_m)$ are bounded by

$$\|d_{ai}(\bar{x}_m)\| < d_{aiU}, \; i = 1, \ldots, m,$$

where d_{aiU} are unknown.

Assumption 2: There exist known constants $g_{iU} > 0$ such that $\|g_i(\cdot)\| \le g_{iU} \;\; \forall i = 1, \ldots, m-1$.

The controller design for the general plant (1) follows the following steps.

Step 1: Let $e = z_1 = x_1 - x_{1d} = x_1 - y_m$ be the model output tracking error. The following inequality holds:

$$|g_1 d_{a1}| \le K_1^* \varphi_1,$$

where $K_1^* = g_{1U} d_{a1U}$ and $\varphi_1 = 1$. Since K_1^* is not known, it is estimated by $\hat{K}_1$ with an estimated error $\tilde{K}_1 = \hat{K}_1 - K_1^*$.

Choose a virtual control input

$$x_{2d} = -g_1^{-1}[c_1 z_1 + f_1 - \dot{x}_{1d} - u_{2dvsc}], \tag{2}$$

where

$$u_{2dvsc} = -\hat{K}_1 \varphi_1 \, \mathrm{sgn}(z_1) \tag{3}$$

is the variable structure controller. The estimate $\hat{K}_1$ is updated according to an update law:

$$\dot{\hat{K}}_1 = \dot{\tilde{K}}_1 = \Gamma_{k1} \varphi_1 |z_1|, \tag{4}$$

where $\Gamma_{k1} > 0$ is a design constant.

The tracking error dynamics become

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{x}_{1d} \\ &= f_1 + g_1(x_2 + d_{a1}) - \dot{x}_{1d} \\ &= -c_1 z_1 - \hat{K}_1 \varphi_1 \, \mathrm{sgn}(z_1) + g_1 d_{a1} + g_1(x_2 - x_{2d}). \end{aligned}$$

Letting a Lyapunov function be

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{K}_1^2 \Gamma_{k1}^{-1},$$

its derivative is given by

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1 + \tilde{K}_1\Gamma_{k1}^{-1}\dot{\tilde{K}}_1\\ &= -c_1z_1^2 - \hat{K}_1\varphi_1|z_1| + z_1g_1d_{a1} + z_1g_1z_2 + \tilde{K}_1\varphi_1|z_1|\\ &\le -c_1z_1^2 - \hat{K}_1\varphi_1|z_1| + K_1^*\varphi_1|z_1| + z_1g_{1U}z_2 + \tilde{K}_1\varphi_1|z_1|\\ &= -c_1z_1^2 + z_1g_{1U}z_2,\end{aligned}$$

where $z_2 = x_2 - x_{2d}$. The term $z_1g_{1U}z_2$ will be cancelled in the next step.

Step i: $(2 \le i < m)$ Let $z_{i+1} = x_{i+1} - x_{(i+1)d}$, $2 \le i \le m-1$, be the tracking error. Similar derivation to that of Step 1 can be used with virtual control input

$$x_{(i+1)d} = -g_i^{-1}[g_{(i-1)U}z_{i-1} + c_iz_i + f_i - \dot{x}_{id} - u_{(i+1)dvsc}], \quad (5)$$

variable structure control

$$u_{(i+1)dvsc} = -\hat{K}_i\varphi_i \ \operatorname{sgn}(z_i), \quad (6)$$

update law

$$\dot{\hat{K}}_i = \dot{\tilde{K}}_i = \Gamma_{ki}\varphi_i|z_i|, \quad (7)$$

and Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{K}_i^2\Gamma_{ki}^{-1},$$

to obtain the derivative of the Lyapunov function as

$$\dot{V}_i \le \left(\sum_{k=1}^{i} -c_kz_k^2\right) + z_ig_{iU}z_{i+1}.$$

Step m:

This is the last step. Let $z_m = x_m - x_{md}$ be the tracking error. The control effort is selected as

$$u_c = -g_m^{-1}[g_{(m-1)U}z_{m-1} + c_mz_m + f_m - \dot{x}_{md} - u_{(m+1)dvsc} + g_mr_s]. \quad (8)$$

Note the addition of the last term g_mr_s in the u_c expression to compensate with the shaped reference r_s. With similar variable structure control

$$u_{(m+1)dvsc} = -\hat{K}_m\varphi_m \ \operatorname{sgn}(z_m), \quad (9)$$

update law

$$\dot{\hat{K}}_m = \dot{\tilde{K}}_m = \Gamma_{km}\varphi_m|z_m|, \quad (10)$$

and Lyapunov function

$$V_m = V_{m-1} + \frac{1}{2}z_m^2 + \frac{1}{2}\tilde{K}_m^2\Gamma_{km}^{-1},$$

the derivative of the Lyapunov function becomes

$$\dot{V}_m \le \left(\sum_{k=1}^{m} -c_kz_k^2\right).$$

Therefore, if $c_k > 0, \ \forall k = 1,\ldots,m$, $\dot{V}_m$ is negative semi-definite or $\dot{V}_m \le 0$. From Theorem 4.1 in [14], the zero equilibrium points of the error dynamics are stable.

III. Flexible-Joint Robot Manipulator

Joint flexibility exists in most robot manipulators, arising from driving components such as actuators, gear teeth, and transmission belts or from joint absorber to reduce impact force and damage [12]. Controlling the flexible-joint robot is complicated because of its complicated mathematical model and because its number of degrees of freedom is twice the number of control inputs, the so-called under actuation.

In this paper, a flexible-joint robot, shown in Fig. 2, was used in the experiment. An accelerometer is mounted next to the payload whose location is at the tip. Two optical encoders are used to measure the motor angle θ_2 and the link angle relative to the motor $\theta_1 - \theta_2$. Two soft springs are attached to the link and the block to provide flexibility.

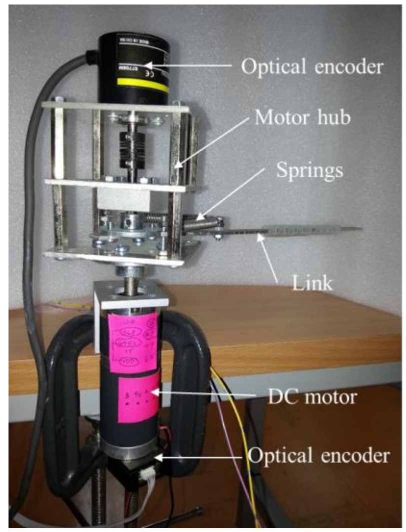

Fig. 2 Flexible-joint robot.

Fig. 3 is a diagram of the experimental set-up. A host computer, with necessary software, is used to communicate with user and a target computer. The target computer contains a data acquisition card whose functions are to acquire sensor signals and to send out actuator command from the control algorithm. The host and target computers are connected to each other via a LAN line. Control signal is sent as voltage to a motor amplifier board to be amplified to a level that can drive the DC motor. An IC chip accelerometer is mounted at the tip to measure linear acceleration. A DC power supply supplies required current to the motor amplifier board.

The flexible-joint robot model can be put in the strict-feedback form (1) as

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= f_2 + g_2x_3,\\ \dot{x}_3 &= x_4,\\ \dot{x}_4 &= f_4 + g_4u,\end{aligned}$$

where

$$f_2 = -\frac{k_{slin}}{(J_l + J_p)}x_1 - \frac{c_1}{(J_l + J_p)}x_2, \ g_2 = \frac{k_{slin}}{(J_l + J_p)},$$

$$f_4 = \frac{k_{slin}}{J_h}x_1 + \frac{c_1}{J_h}x_2 - \frac{k_{slin}}{J_h}x_3 - \left(\frac{k_tk_v}{J_hR} + \frac{c_2}{J_h} + \frac{c_1}{J_h}\right)x_4,$$

$$g_4 = \frac{k_t k_a}{J_h R}, \quad (11)$$

$x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, $x_4 = \dot{\theta}_2$. u is the control command voltage from the data acquisition card. k_{slin} is the linearized spring stiffness. J_l, J_h, and J_p are the link, hub, and payload masses moment of inertia about the pivot point, respectively. c_1 and c_2 are the damping constant at link bearing and at motor bearing. R, k_v, k_t, and k_a are the motor coil resistance, back-EMF constant, current-to-torque gain, and amplifier gain.

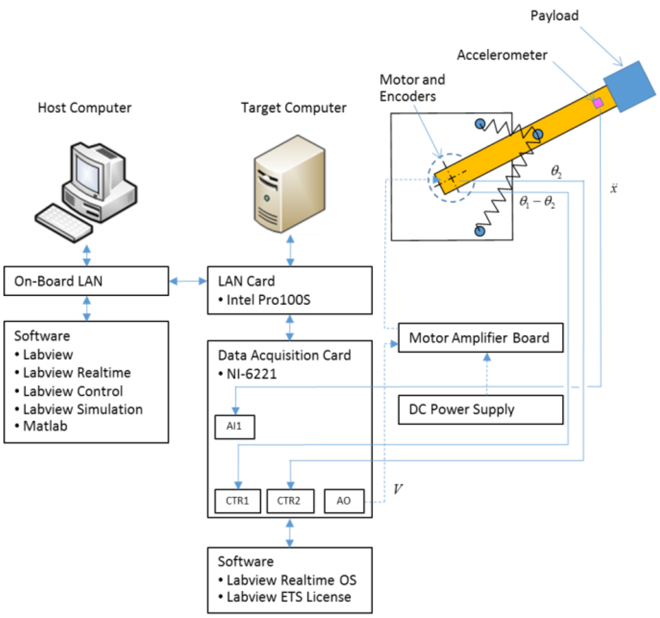

Fig. 3 Diagram of the experimental set-up and associated software and hardware.

Using the model parameters from system identification, the mode parameters, which are the natural frequency and damping ratio, are given by

$$\omega_n = 16.206 \text{ rad/s}, \zeta = 0.052. \quad (12)$$

IV. Simulation and Experimental Results

The proposed backstepping model matching scheme used to produce the results is shown in Fig. 1. The ZV input shaper was designed from the mode parameters of the reference model. The reference model was selected to be

$$\frac{y_m(s)}{r_s(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

where ω_n and ζ are given by (12). The backstepping variable structure controllers and update laws are given by (2)-(10), with the following design parameters:

$$c_i = 10, \Gamma_{ki} = 1, \forall i = 1, ..., 4,$$
$$g_{1U} = 1, g_{2U} = 300, g_{3U} = 1.$$

The functions f_i and g_i in the control laws are given by (11).

For comparison, the ZVDk [2], the extra-insensitive (EI) [15], and the impulse-time perturbation (PEI-IS) [16] input shapers, are used. They are applied to a closed-loop system as shown in Fig. 4. The mapping from the control effort u to the motor angular position θ_2 represents the rigid-body dynamics of the robot. θ_2 is fed back whereas a proportional control gain $K_p = 0.05$ was used throughout. The input shaper is placed outside of the loop and is designed using the mode parameters of the closed-loop system from r_s to θ_1. The allowable percentage vibration is set equal to $V_{\lim} = 0.1$.

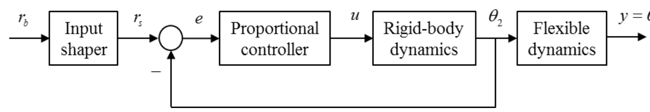

Fig. 4 Closed-loop system with robust input shapers.

A. Larger Amount of Uncertainty Can Be Tolerated

In both simulation and experiment, the baseline reference signal was chosen as a square signal with an amplitude of 1 radian and a period of 15 seconds. The step responses were recorded. Sensitivity curves were obtained using a formula [17]:

$$V(\omega_n, \zeta) = \frac{\max(h_{SG}(t)) - h_G(\infty)}{\max(h_G(t)) - h_G(\infty)} \times 100,$$

where $h_G(t)$ is the step response of the system without the input shaper and $h_{SG}(t)$ is the step response with the input shaper.

Fig. 5 shows the simulation result of the percentage vibration as a function of normalized frequency (ω_a / ω_n), where ω_a is the actual natural frequency and ω_n is the model natural frequency. Among the four robust input shapers, the PEI-IS shaper has the largest insensitivity to uncertainty in natural frequency, especially for higher frequencies, followed by the EI shaper, the ZVD shaper, and the ZV shaper. However, the proposed backstepping model matching input shaper (BMM-IS) has practically zero sensitivity to uncertainty in natural frequency. This is because the shaper was designed from the reference model whose mode parameters are exactly known. Note that, around $\omega_a / \omega_n \approx 2$, V may be over 100% because the actual plant is nonlinear.

Fig. 6 shows a simulation result of the percentage vibration as a function of the model damping ratio ζ. The actual damping ratio is given by (12) as $\zeta = 0.052$. It can be seen that the ZV, EI, and PEI-IS shapers have similar amount of insensitivity to uncertainty in damping ratio. The ZVD shaper has larger insensitivity, and the proposed BMM-IS shaper has practically zero sensitivity. Note that, around $\zeta = 0.052$, PEI-IS and EI shapers have $V \approx 10\,\%$ because their $V_{\lim} = 0.1$.

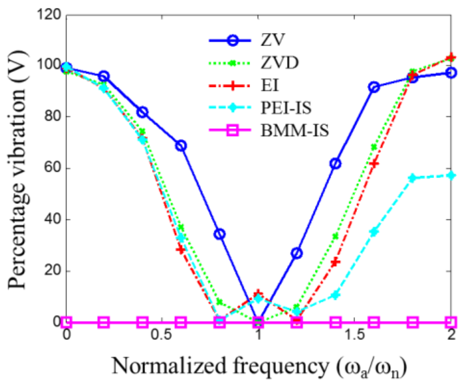

Fig. 5 Simulation result. Percentage vibration as a function of normalized frequency.

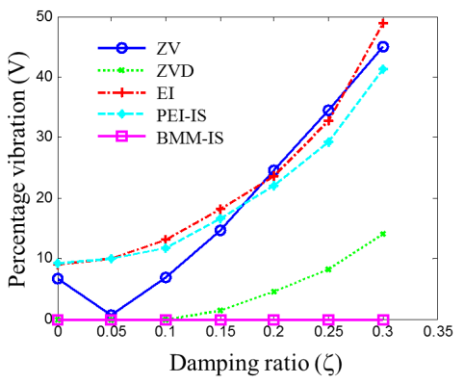

Fig. 6 Simulation result. Percentage vibration as a function of damping ratio.

Fig. 7 shows experimental result using the robot shown in Fig. 2. The experimental result was close to the simulation result even though some small vibration appeared due to the imperfection of the mathematical model of the robot.

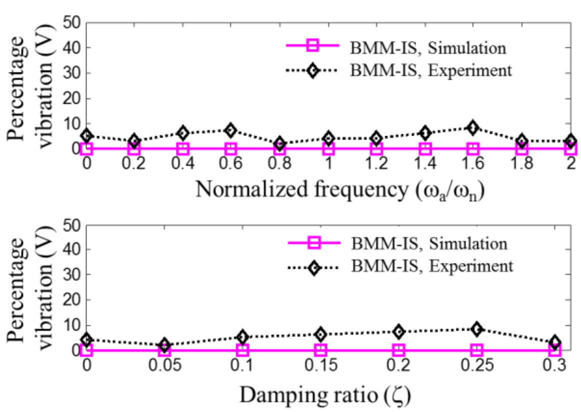

Fig. 7 Experimental result. Percentage vibration as a function of normalized frequency and damping ratio.

B. Shaped Reference Has Shorter Duration

Fig. 8 shows 10% insensitivity to the uncertainty in the natural frequency versus input shaper length for ZVD^k, EI, PEI-IS, and the proposed BMM-IS input shapers. The 10% insensitivity is the width of the sensitivity curve when the percentage vibration (V) is 10%; hence, it is related to robustness of the input shaper. The input shaper length is the time location of the last impulse in the impulse sequence subtracted by that of the first impulse. This length relates directly to the time it takes the shaped reference signal to reach the final value and to how fast the manipulator moves.

From Fig. 8, for the same input shaper length, the PEI-IS shaper is more robust to the uncertainty in the natural frequency than the EI shaper whereas the ZVD^k shaper is the least robust. With ZVD^k, EI, and PEI-IS shapers, the input shaper length increases with the insensitivity, which means the manipulator moves slower when the input shaper becomes more robust. However, for the proposed BMM-IS, the input shaper length is that of the ZV shaper and the length does not increase with the insensitivity. This is because the controller matches the closed-loop system to fixed reference model, and the ZV shaper is designed from that model.

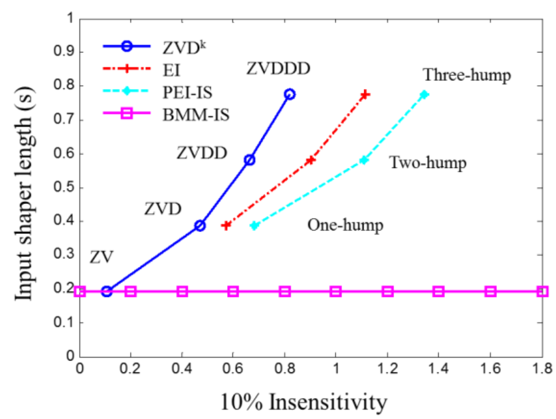

Fig. 8 10% insensitivity versus input shaper length.

Other percentage insensitivity as well as the insensitivity to the uncertainty in the damping ratio or other plant parameters give analogous result to that of Fig. 8.

C. Applicable to Nonlinear System

For the destructive interference of the impulse responses to apply, the input shaping technique requires the plant to be linear. If the plant is nonlinear, it must be linearized, and the performance of the input shaper may degrade. However, for the proposed backstepping model matching input shaper, the feedback control handles the nonlinearity, and the performance of the input shaper will not depend on the nonlinearity in the plant.

Backlash is a type of hard nonlinearities that has been known to cause residual vibration [12] due to its hysteresis effect on the control input. In simulation, backlash is given by

$$\dot{v} = \begin{cases} \dot{u}, & \begin{array}{l}\text{if } \dot{u} > 0 \text{ and } v = (u - b^{+}) \text{ or} \\ \text{if } \dot{u} < 0 \text{ and } v = (u - b^{-})\end{array} \\ 0, & \text{otherwise} \end{cases},$$

where v is the command given to the model, b^{+} and b^{-} are positive and negative constants.

Because the input shaper is designed based on the assumption that the plant is linear, it cannot suppress the vibration induced by backlash. Assuming perfect model, Fig. 9(Top) shows the existence of the residual vibration in the link position θ_1 when the ZV input shaper was used with a backlash parameters $b^+ = 0.5$ and $b^- = -0.5$. However, when the proposed BMM-IS shaper was used, the residual vibration induced by backlash was totally suppressed. Fig. 9(Bottom) shows the increase in the percentage vibration when the amount of backlash is increased using the ZV input shaper. However, the percentage vibration remains zero with the proposed BMM-IS shaper.

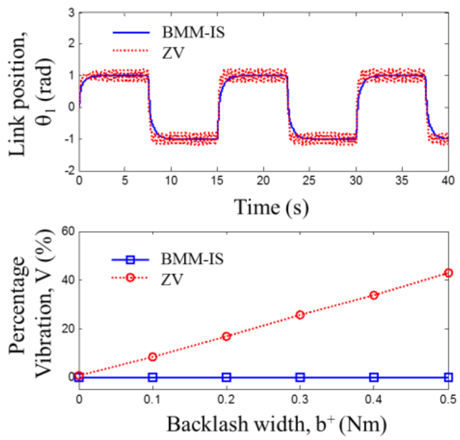

Fig. 9 (Top) Link position under the influence of backlash $b^+ = 0.5$ Nm. (Bottom) Residual vibration as a function of backlash width b^+.

D. Disturbance Rejection

Disturbance can induce flexible modes causing vibrations. In the proposed BMM-IS system, the plant-input disturbance can be included in the term $d_{am}(\bar{x}_m)$ of (1). The backstepping variable structure controller will ensure stability of the states in the presence of $d_{am}(\bar{x}_m)$.

Fig. 10 shows a simulation result when a plant-input disturbance $d_{a4} = \bar{A}\sin(2\pi t)$ was added to the control input $u(t)$. The amplitude $\bar{A}$ ranges from 0.004 to 0.010 volts. The percentage residual vibration, induced by plant-input disturbance, is lower and increases in a slower rate when the BMM-IS shaper was used.

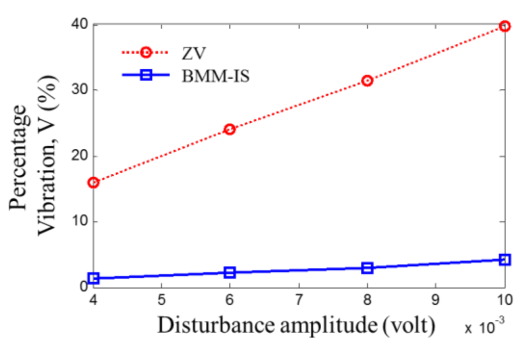

Fig. 10 Residual vibration amount when sinusoidal disturbances with various amplitudes are present.

V. Conclusions

An input shaping system with backstepping and variable structure controllers is presented. The controllers match the closed-loop system to a reference model so that the input shaper can be designed using the mode parameters of the reference model, which are accurate. As a result, substantially large amount of uncertainty can be tolerated. The ZV input shaper used in the system has the shortest duration and the duration does not increase with insensitivity. The proposed system is also applicable to nonlinear system because the nonlinear system is matched to a linear reference model, which is eligible to input shaping. The controllers also reject disturbance, reducing the disturbance-induced vibration.

Future work includes on-line system identification using intelligent systems so that the algorithm will be model-independent and adaptive, comparison with adaptive input shaping schemes, and further investigation on multi-mode systems.

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