

Robust Model Predictive Controller Design for Thermal Plate System via Linear Matrix Inequality Approach

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Abstract

The design of model predictive controller is based on mathematic model of the system around an operating point. When operating condition is changed to another point, the performance of model predictive controller may be poor because parameters uncertainty of the system. For dealing with plant uncertainty, this paper focuses on the design of robust model predictive control via linear matrix inequality and compares it with conventional model predictive controller. A thermal plate system is used as the demo of uncertain plant. Experimental results show that the proposed method handles uncertainty better than conventional model predictive control, when the system is operated in a wide range of operating conditions.

Keywords: Robust model predictive control, Linear matrix inequality, Thermal plate system.

1. Introduction

Thermal plate or thermoelectric module is a kind of heat pump device, when DC voltage is applied to device the substrate surface become cold because positive and negative semiconductor pellets array absorb heat energy and the another substrate surface become hot for releasing the heat energy, as shown in Fig 1 [8].

Thermal plate technology is applied to many wide applications such as, small laser diode coolers, portable refrigerators, small heater device,

heat exchanger, liquid coolers and scientific thermal conditioning.

Several techniques have been proposed in the literature for controlling a temperature of a thermal plate, for example, robust control, fuzzy control, model output following control, model reference adaptive control and model predictive control in difference applications.

In [2-4] have proposed the fuzzy control of the thermal plate system for dealing with nonlinear behavior of a thermal plate such as,

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thermal mass, ambient temperature and cooling load of thermal plate device.

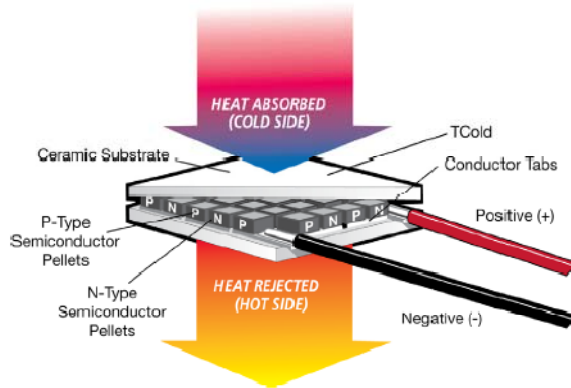


Fig .1 Thermal plate device.

In [5] model output following control has proposed for dealing with mismatch uncertainty of plant model by feed-forward controller and model reference adaptive control has proposed in [8] for calculating a new optimal control effort when plant parameters has changed.

Controller based on robust control, such as a disturbance observer method [6], robust right co-prime factorization and pre-compensator [7] have been proposed for controlling thermal plate system. In general, robust control is designed based on plant parameters and uncertain parameters for synthesizing robust controller to deal with disturbance signal and parameters varying of plant model.

Model predictive control (MPC) is a modern powerful control strategy in industry and process control. MPC is a form of control in which the current control action is obtained by solving a finite-time constrained optimization online to minimize future tracking error. In [9] model predictive control has been proposed in thermal

plate water cooler for saving energy consumption by optimal cost function of control effort.

From the literature, many researchers developed control algorithms to deal with plant uncertainty and reject a disturbance. As a result, robust model predictive control should be implemented because this algorithm has the ability to deal with plant model uncertainty when operating condition of a thermal plate is changed.

In this paper, temperature control of thermal plate system by using linear matrix inequality for synthesizing robust model predictive controller from numerous plant model parameters is proposed. The proposed strategy is verified in both simulation and experiment tests. A comparison between robust model predictive control and simple model predictive control, based on tracking performance from numerous working conditions, is presented.

The paper is organized as follows. Firstly, mathematic model based on thermal plate system is described. Secondly, the hardware of thermal plate system is set up. Thirdly, system identifier for finding plant parameters by least square method is presented in this section. Next, model predictive control and robust model predictive control via linear matrix inequality are described in this section. Further, experiment result is given to confirm the proposed method. Finally, the conclusion of this work is presented in the final section.

2. Mathematic model of thermal plate

This work use conservation of energy [1] as shown in Eq. (1) for developing system model.

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$$C \frac{dT}{dt} = u - \frac{1}{R}T \quad (1)$$

$$C_n = \frac{\sum_{i=1}^N C_i}{N} \quad (6)$$

Where:

T is surface temperature of thermal plate (°C)

C is thermal capacitance of thermal plate (J/K)

R is thermal resistant of thermal plate (°C/W)

u is control input signal of thermal plate

The state-space representation of the nominal plant can be derived from Eq. (1) and can be shown in Eq. (2).

$$\begin{cases} x_m(k+1) = A_n x(k) + B_n u(k) \\ y(k) = C_n x(k) \end{cases} \quad (2)$$

Where the state-space vectors are $x_m = [T]$ and the state-space matrix are $A_n = \left[-\frac{1}{RC}\right]$, $B_n = \left[\frac{1}{C}\right]$, $C_n = [1]$ and the plant matrices are set of working condition, when the set (Ω) are

$$\left\{ \begin{bmatrix} A_1 & B_1 \\ C_1 & 0 \end{bmatrix}, \dots, \begin{bmatrix} A_N & B_N \\ C_N & 0 \end{bmatrix} \right\} \quad (3)$$

Therefore, nominal plant matrices can be calculated by Eqs. (4) – (6).

$$A_n = \frac{\sum_{i=1}^N A_i}{N} \quad (4)$$

$$B_n = \frac{\sum_{i=1}^N B_i}{N} \quad (5)$$

When N is the total number of operating condition of the system and matrix subscripts i is plant parameters on working condition i . The plant parameters of thermal plant system are varying due to operating at different working condition. The essential parameters of the thermal plate can be extracted from open-loop experimental by system identification such as, least square algorithm, open-loop step test at every working condition.

3. Hardware setup

This work use numerous equipments for setting the experiment such as, power device used to distribute 12 Volt DC from power supply to the thermal plate when control signal from data acquisition (Arduino MEGA 2560) is applied to power device in a finite time, thermocouple type K for measuring the dynamic of temperature on a thermal plate surface, IC AD595 for amplifying the temperature signal from thermocouple and low-pass filter for reducing the amplitude of signal. The amplified of a temperature signal is used to feedback data for calculating of an error signal in the controller section. In this study Matlab and Simulink software used to written the algorithm program of open-loop step test for finding system parameters and written the different control algorithms, for example, MPC, state-feedback and RMPC for studying the ability of controllers in thermal plate system. The block diagram of each equipment can be seen in Fig. 2.

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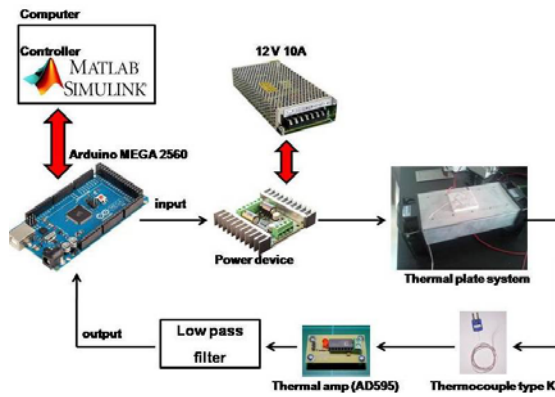


Fig. 2 Block diagram of hardware and software configuration.

4. System identification

Essential parameters of the system can be determined by least square algorithm, which is a process to find unknown parameters from a physical model, the algorithm can be written as

$$Y = \Phi\theta \quad (7)$$

Where:

θ is an unknown parameter vector, such as a model parameter.

Φ is known regression matrix, which is the matrix contain the data of state and input, from experiment condition.

Y is known measurements vector, such as output of the system.

The solution of θ can be found from Eq. (8).

$$\theta = \Phi^{-1}Y \quad (8)$$

The solution of least square θ_{LS} can be written as

$$\theta_{LS} = (\Phi^T \Phi)^{-1} \Phi^T Y. \quad (9)$$

Mathematic model from Eq. (1) can be rewritten in the form of least square algorithm as

$$\begin{matrix} \dot{T} \\ Y \end{matrix} = \underbrace{\begin{bmatrix} T & u \end{bmatrix}}_{\varphi} \begin{bmatrix} -\frac{1}{RC} \\ \frac{1}{C} \end{bmatrix}. \quad (10)$$

In order to find θ using least square algorithm this method need to log input and output data. Therefore, this system will be discrete and apply Euler forward method to differential term as

$$\underbrace{\frac{T_{k+1}-T_k}{T_s}}_Y = \underbrace{\begin{bmatrix} T & u \end{bmatrix}}_{\varphi} \begin{bmatrix} -\frac{1}{RC} \\ \frac{1}{C} \end{bmatrix}. \quad (11)$$

Where T_s is the sampling time for logging data. This work set sampling time equal to 0.05 s. The open-loop test is used to identified two essential parameters as shown in Fig. 3. The identifier process use input signal condition as a constant signal between 0 and 255 ,when 255 is mean 100 % of PWM duty cycle or 5V Dc, send to thermal plate system. Plant parameters can be extracted by identification process from recorded input-output data. The plant parameters are varying due to numerous input conditions as shown in Table 1. The first column and second column of table 1 are represented steady-state value of input and output signal, respectively. The last two columns are represented plant parameters in discrete time system with 0.05 s of sampling time. The nominal parameters of system

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parameters can be calculated from Eq. (4) and (5), respectively.

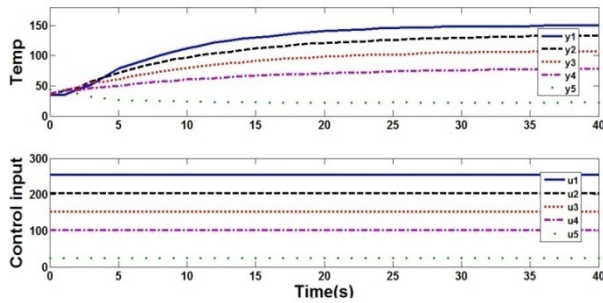


Fig. 3 Output and input of open-loop step test for each operating condition.

Table. 1 plant parameters for each input condition

Input condition		Plant parameters	
Steady-state input	Steady-state output (°C)	A_n	B_n
255	155	0.9954	0.0128
204	138	0.9952	0.0132
153	112	0.9948	0.0137
102	81	0.9939	0.0147
25.5	24	0.9663	0.0128
Nominal parameters		0.9892	0.0134

4.1 Validation of plant model

The percentage of best fit criterion of the model is used to check an accurate of model from system identification method. This work use best fit criterion according to [10] as shown in Eq. (12).

$$Best\ Fit = \left(1 - \frac{\sum_{i=1}^N |y_i - \hat{y}_i|}{\sum_{i=1}^N |y_i - \bar{y}_i|} \right) \times 100\% \quad (12)$$

Where N is a total number of data, y_i is a measurement output at i time step and \hat{y}_i is a simulation output from the model at i time step .

The best fit of each model can be shown in table. 2.

Table. 2 The best fit of each model

Model	Steady-state input	Steady-state output (°C)	Best Fit
1	255	155	87.9%
2	204	138	86.8%
3	153	122	87.4%
4	102	81	88.5%
5	25.5	24	85.8%

The example of best fit criterion for the first model can be seen in Fig. 4.

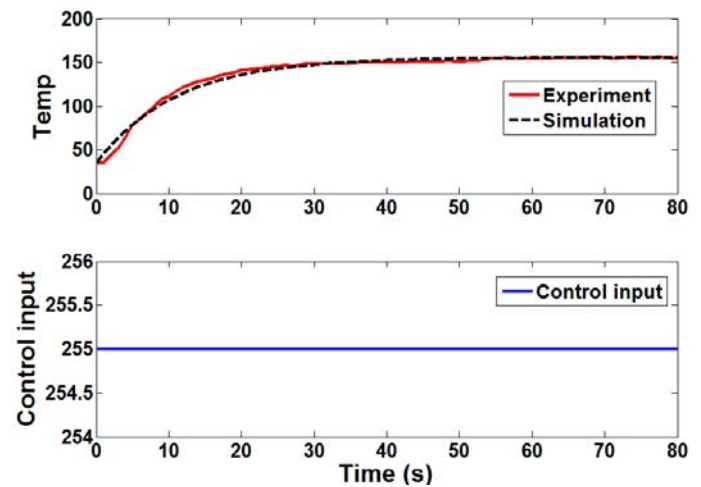


Fig. 4 The comparison of the real output and simulated output generated by model 1.

5. Controller design

In this work, model predictive control and robust model predictive control algorithm are used to control thermal plate system for studying performance of difference controllers.

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5.1 Model predictive controller design

According to [11] the designs of model predictive control using the difference of state-space form so, taking a difference operation on both sides of Eq. (2). The difference of state-space form can be shown in Eq. (13).

$$\Delta x_m(k+1) = A_n \Delta x_m(k) + B_n \Delta u(k) \quad (13)$$

Where:

$$\Delta x_m(k+1) = x_m(k+1) - x_m(k)$$

$$\Delta x_m(k) = x_m(k) - x_m(k-1)$$

$$\Delta u(k) = u(k) - u(k-1)$$

The input to state-space model is $\Delta u(k)$. The next step is to connect $\Delta x_m(k)$ to output $y(k)$.

Therefore, define a new state variable vector is chosen be

$$x(k) = [\Delta x_m(k)^T \quad y(k)^T]^T. \quad (14)$$

The difference of output equation can be shown as

$$y(k+1) - y(k) = C_n \Delta x_m(k+1). \quad (15)$$

The augmented model, which is a state-space model with embedded integrator, will be used in the design of model predictive control by putting together Eq. (13) and Eq. (15) leads to following state-space model as shown in Eq. (16).

$$\begin{cases} \begin{matrix} x(k+1) \\ \left[\begin{matrix} \Delta x_m(k+1) \\ y(k+1) \end{matrix} \right] \end{matrix} = \begin{matrix} A \\ \left[\begin{matrix} A_m & o_m^T \\ C_m A_m & 1 \end{matrix} \right] \end{matrix} \begin{matrix} x(k) \\ \left[\begin{matrix} \Delta x_m(k) \\ y(k) \end{matrix} \right] \end{matrix} + \begin{matrix} B \\ \left[\begin{matrix} B_m \\ C_m B_m \end{matrix} \right] \end{matrix} \Delta u(k) \\ y(k) = \begin{matrix} C \\ \left[\begin{matrix} o_m & 1 \end{matrix} \right] \end{matrix} \begin{matrix} \Delta x_m(k) \\ y(k) \end{matrix} \end{cases} \quad (16)$$

The future control trajectory is denoted by Eq. (17).

$$\Delta u(k_i), \Delta u(k_i+1), \dots, \Delta u(k_i+N_c-1) \quad (17)$$

Where N_c is the control horizon, which is the number of parameters used to capture the future control trajectory, and the future state variables are predicted for N_p , where N_p is called the prediction horizon. This work denote the future state variables as

$$x(k_i+1|k_i), x(k_i+2|k_i), x(k_i+m|k_i), \dots, x(k_i+N_p|k_i). \quad (18)$$

Where $x(k_i+m|k_i)$ is the predicted state variable at k_i+m with given current plant information $x(k_i)$. The control horizon N_c is chosen to be less than or equal to the prediction horizon.

Based on the state-space model A, B, C , the future state variables are calculated sequentially using the set of future control parameters as

$$x(k_i+1|k_i) = Ax(k_i) + B\Delta u(k_i)$$

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$$\begin{aligned}
 x(k_i + 2|k_i) &= A(k_i + 1|k_i) + B\Delta u(k_i + 1) \\
 &= A^2x(k_i) + AB\Delta u(k_i) + \\
 &\quad B\Delta u(k_i + 1) \\
 &\quad \vdots \\
 &\quad \vdots \\
 x(k_i + N_p|k_i) &= A^{N_p}x(k_i) + A^{N_p-1}B\Delta u(k_i) \\
 &\quad + A^{N_p-2}B\Delta u(k_i + 1) + \dots + \\
 &\quad + A^{N_p-N_c}B\Delta u(k_i + N_c - 1). \quad (19)
 \end{aligned}$$

The set of predicted output variables is calculated sequentially as shown in Eq. (20).

$$\begin{aligned}
 y(k_i + 1|k_i) &= CAx(k_i) + CB\Delta u(k_i) \\
 y(k_i + 2|k_i) &= CA^2x(k_i) + CAB\Delta u(k_i) \\
 &\quad + CB\Delta u(k_i + 1) \\
 y(k_i + 3|k_i) &= CA^3x(k_i) + CA^2B\Delta u(k_i) \\
 &\quad + CAB\Delta u(k_i + 1) \\
 &\quad + CB\Delta u(k_i + 2) \\
 &\quad \vdots \\
 &\quad \vdots \\
 y(k_i + N_p|k_i) &= CA^{N_p}x(k_i) \\
 &\quad + CA^{N_p-1}B\Delta u(k_i) \\
 &\quad + CA^{N_p-2}B\Delta u(k_i + 1) + \dots + \\
 &\quad + CA^{N_p-N_c}B\Delta u(k_i + N_c - 1). \quad (20)
 \end{aligned}$$

Define vectors X and U as Eq.(21) and Eq.(22).

$$X = \begin{bmatrix} y(k_i + 1|k_i) & y(k_i + 2|k_i) & \dots \\ & y(k_i + N_p|k_i) & \end{bmatrix}^T \quad (21)$$

$$U = \begin{bmatrix} \Delta u(k_i) & \Delta u(k_i + 1) & \dots \\ & \Delta u(k_i + N_c - 1) & \end{bmatrix}^T \quad (22)$$

The dimension of X is N_p and the dimension of U is N_c . Eq. (20) can be rewritten in a compact matrix form as

$$X(\bar{k}) = FX(\bar{k}) + \phi U(\bar{k}) \quad (23)$$

Where:

$$X(\bar{k}) = \begin{bmatrix} x(\bar{k} + 1|k) \\ x(\bar{k} + 2|k) \\ \vdots \\ x(\bar{k} + N_p - 1|k) \\ x(\bar{k} + N_p|k) \end{bmatrix},$$

$$U(\bar{k}) = \begin{bmatrix} \Delta u(\bar{k} + 1|k) \\ \Delta u(\bar{k} + 2|k) \\ \vdots \\ \Delta u(\bar{k} + N_c - 1|k) \\ \Delta u(\bar{k} + N_c|k) \end{bmatrix}, F = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N_p} \end{bmatrix}$$

$$\phi = \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots & CA^{N_p-N_c}B \end{bmatrix}$$

Define cost function as Eq. (24).

$$\bar{J}(x(\bar{k}), U(\bar{k})) = X^T(\bar{k})QX(\bar{k}) + U^T(\bar{k})\mathcal{R}U(\bar{k}) \quad (24)$$

Where: $Q = r_Q[I_{N_p \times N_p}]$ and $\mathcal{R} = r_{\mathcal{R}}[I_{N_c \times N_c}]$

Substitution Eq. (23) into Eq.(24) and find the optimal U by using the first derivative as shown in Eq. (25).

$$\frac{\partial \bar{J}(x(\bar{k}), U(\bar{k}))}{\partial U} = 2\phi^T QFX(\bar{k}) + 2(\phi^T Q\phi + \mathcal{R})U(\bar{k}) \quad (25)$$

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The optimal solution for control signal can be determined by set the first derivative to zero, so optimal control signal can be shown in Eq. (26).

$$U(\bar{k}) = -(\phi^T Q \phi + \mathcal{R})^{-1} \phi^T Q F X(\bar{k}) \quad (26)$$

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The state feedback in the form of model predictive control can be determined by Eq. (27).

$$K_{mpc} = \overbrace{[1 \ 0 \ \dots \ 0]}^{N_c} (\phi^T Q \phi + \mathcal{R})^{-1} \phi^T Q F \quad (27)$$

The state feedback gain from Eq. (27) used to implement of model predictive control for the temperature control of thermal plate system.

However, the model used for prediction has at least one eigenvalue on the unit circle. As a result, it inherits a numerical instability problem when the prediction horizon (N_p) becomes large [11].

Stability cannot be guaranteed with a small prediction horizon and control horizon parameters.

Therefore, this work set a prediction horizon (N_p) to 10 and control horizon (N_c) has been chosen to 5 for a better process respond.

The value of N_p and N_c have chosen from simulation by using MPC from Eq. (27) as a controller in thermal plate system.

5.2 Robust model predictive control via linear matrix inequality

The design of MPC using nominal parameters from table 1, but the RMPC using each parameter in table 1 for synthesizing a robust controller gain. As a result, RMPC has ability to handle the variation of model parameters.

The RMPC synthesis by first deriving an upper bound on the robust performance from uncertain plant in Eq. (3), then minimize this upper bound with a MPC state feedback control law [12] as shown in Eq. (28) and the close-loop system can be shown in Eq. (29).

$$u(k) = K_{RMPC} x(k) \quad (28)$$

$$x(k+1) = (A_i + B_i K_{RMPC}) x(k) \quad (29)$$

Consider a quadratic function $V(x(k)) = x(k)^T P x(k)$, $P > 0$ of the state in Eq. (2) with $V(x(0)) = 0$. The decreasing of function V must satisfy the inequality as shown in Eq. (30) for any uncertain set in Eq. (3).

$$\begin{aligned} V(x(k+1)) - V(x(k)) \\ \leq -x(k)^T Q x(k) - u(k)^T R u(k) \end{aligned} \quad (30)$$

Substitution Eq. (28), (29) into Eq.(30) and rearrange in form of Eq. (31).

$$\begin{aligned} (A_i + B_i K_{RMPC})^T P (A_i + B_i K_{RMPC}) - P \\ + K_{RMPC}^T R K_{RMPC} + Q \leq 0 \end{aligned} \quad (31)$$

Pre and post multiplied by X to Eq. (31), then substitution $P = \gamma X^{-1}$ and define $Y = K_{RMPC} X$ and multiplied with $-1/\gamma$, so Eq.(31) become Eq.(32) as

$$\begin{aligned} -X\gamma + (A_i X + B_i Y)^T (\gamma X^{-1}) (A_i X + B_i Y) \\ + Y^T R Y + X Q X \leq 0 \end{aligned} \quad (32)$$

The linear matrix inequality as

$$\begin{bmatrix} X & 0 & 0 & X A_i + Y B_i \\ 0 & \gamma I & 0 & Q^{1/2} X \\ 0 & 0 & \gamma I & R^{1/2} Y \\ X A_i^T + Y^T B_i^T & X Q^{T/2} & Y^T R^{T/2} & X \end{bmatrix} \geq 0 \quad (33)$$

For every $i = 1, 2, \dots, N$, when N is a total number of operating condition. The linear matrix

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inequality from Eq. (33) can be solved by CVX toolbox [13] for minimization parameter γ and finding optimal parameters X and Y . The Lyapunov matrix and RMPC state feedback gain can be calculated from Eq.(34) and Eq. (35), respectively.

$$P = X^{-1} \quad (34)$$

$$K_{RMPC} = YX^{-1} \quad (35)$$

This work implementations MPC and RMPC in form of state feedback as Eq.(27) and Eq. (35) to thermal plate system for studying performance of difference controllers, when working condition is varied.

6. Experimental study

The MPC and RMPC have been implemented as discrete time with sampling time 0.05 s.

The block diagram of control strategy can be shown in Fig. 5.

This work set K_I to 0.04 for tracking performance in steady-state condition of process.

The MPC gain from Eq.(27) and RMPC gain from Eq.(35) have implemented in form of state feedback gain as shown in Fig. 5.

This work uses six operating conditions to verify the performance of difference controllers in term of rise time, overshoot, setting time, steady-state error. Table. 3 show the controller performance and Fig. 6 show the experimental of this work.

Table. 3 the performance of difference controllers

controller	Rise time (S)	% Over Shoot (%)	Setting-Time (S)	SS error
MPC	3.9	5.2	8.3	± 0.315
RMPC	4.2	1.4	6.4	± 0.152

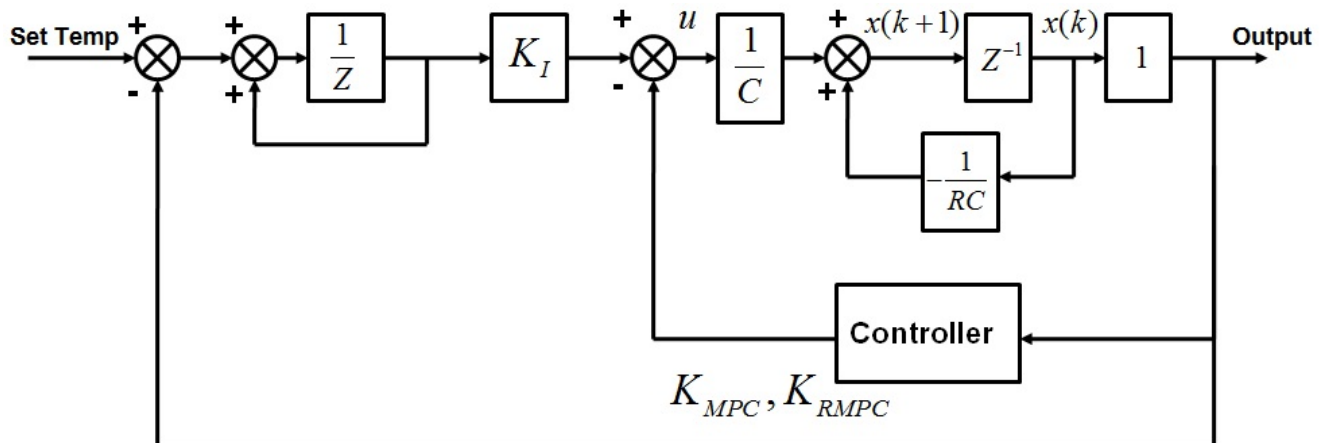


Fig. 5 Block diagram of MPC and RMPC control strategy.

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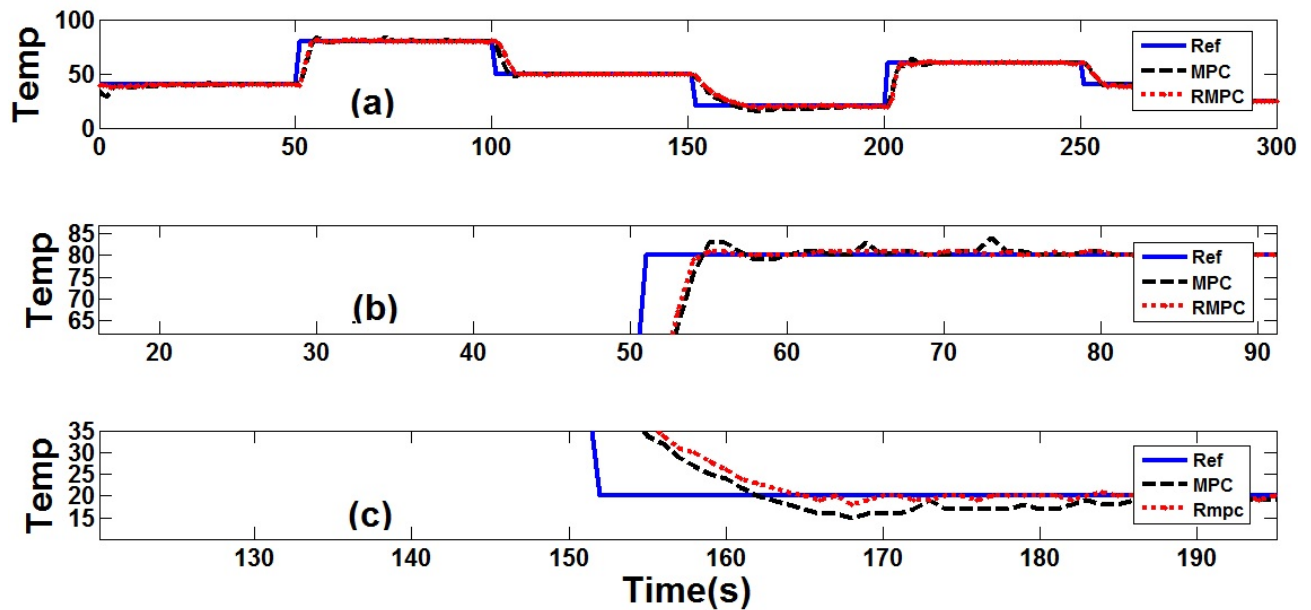


Fig. 6 Experiment result of MPC (dash-line), RMPC (dot-line), when reference signal (solid-line) is varied with case (a) is an overall output respond, case (b) is an output respond at 80 °C working condition, case (c) is an output respond at 20 °C working condition.

From Fig. 6 (a), the tracking performance of temperature output for thermal plate system with RMPC is better than MPC in term of percent maximum overshoot, setting time and steady-state error. However, MPC controller is better than RMPC in term of rise time in all operating conditions.

From Fig. 6 (b), the performance of MPC is similar to RMPC because, the nominal model has plant parameters close to model of the 80 °C operating condition as shown in table. 1.

For cooling situation, the performance of RMPC is better than MPC in term of percent maximum overshoot, setting time and steady-state error as shown in Fig. 6 (c).

7. Conclusion

Experimental results show that the performance of proposed controller is better than

MPC in all of operating condition, where MPC is designed based on the nominal model and RMPC is designed based on variation of plant model via linear matrix inequality approach.

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