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Analysis of Closed-Form Velocity Command that Avoids Violating the Acceleration Limit in Input-Shaped System

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Abstract

Acceleration limit is common in practical actuators. Input shaping is a technique to reduce residual vibration by destructive interference of impulse responses, that is, an impulse response can be cancelled by another impulse response, given appropriate impulse amplitudes and applied times. In application, the input shaper is placed in front of the acceleration limit, followed by the flexible plant. As a result, input-shaped velocity command will be altered by the acceleration limit; therefore, the performance of the input shaper to reduce the residual vibration of the flexible plant will degrade. Previous work relies on optimization routine to obtain a shaper that will not violate the acceleration limit. However, an additional constraint on the impulse time locations is required which limits the achievable performance of the shaper. In this paper, for the first time, a baseline velocity command is found in closed form for an existing input shaper. The shaped velocity command, obtained from passing the baseline velocity command to the input shaper, will not violate the acceleration limit. The input shaper, therefore, can perform well as designed, unaffected by the acceleration limit. Simulation results show the effectiveness of this proposed technique.

***Keywords**:* Input shaping; Vibration reduction; Acceleration limit; Velocity command.

1. Introduction

Input shaping is a technique to reduce residual vibration. The technique is based on destructive interference of impulse responses, that is, an impulse response can be cancelled by another impulse response, given appropriate impulse amplitudes and applied times.

Acceleration limit exists in most actuators in practice. The shaped command can be distorted by the acceleration limit, causing degradation in vibration reduction performance of the input shaper.

Existing researches on input shaping under acceleration limit are very few. Ref. [1] and [2] studied input shapers under velocity command and acceleration upper limit.

They proposed an additional constraint during the design of the unity-magnitude (UM) input shaping system in Fig. 1. This additional constraint ensures that the resulting shaped velocity command will not violate the acceleration limit. The additional constraint is given by

$$v_f = a\left[(t_2 - t_1) + (t_4 - t_3)\right],$$

where v_f is the desired final velocity and a is the acceleration limit.

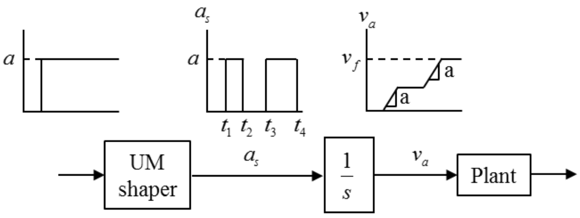

Fig. 1 UM shaper design to avoid violating acceleration limit.

However, this proposed technique requires an optimization routine, which may complicate the design. Moreover, the additional constraint may limit the performance of the input shaper in suppressing the vibration.

In this paper, the baseline command, which is the command input to the input shaper, is modified such that the shaped command will not violate the acceleration limit.

The proposed technique is simple. It will work with any type of input shapers. It is given in closed form so that it does not require optimization routine.

Simulation with a two-mass rigid-flexible system, which represents majority of the flexible systems in practice, shows the effectiveness of the proposed technique in suppressing the residual vibration in the presence of the acceleration limit.

This paper is organized in this way. Section 2 contains derivation of transfer function representing the two-mass rigid-flexible system. Section 3 presents the ZVD input shaper to be used in this work and shows an example of performance degradation of the input shaper from acceleration limit. Section 4 designs a modified baseline velocity command that avoids violating the acceleration limit. Conclusions are given in Section 5.

2. Two-Mass Rigid-Flexible System

Consider a two-mass rigid-flexible system in Fig. 2. In general, the system represents two entities, connected via a flexible part, which encompasses a large majority of actual rigid-flexible systems. The driving one has an absolute position and mass of x_0 and m_0, and the driven one has x_1 and m_1. k, c_0, and c are spring stiffness and two damping constants. f is the control force. The objective is to move both masses from the origin to a displacement X with zero residual vibrations and in a shortest time possible T, that is,

$$\begin{Bmatrix} x_0 \\ x_1 \end{Bmatrix}\Bigg|_{t=T} = \begin{Bmatrix} X \\ X \end{Bmatrix}, \quad \begin{Bmatrix} \dot{x}_0 \\ \dot{x}_1 \end{Bmatrix}\Bigg|_{t=T} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

Gantry crane can be modeled as two-mass rigid-flexible system. The cart mass is m_0. The viscous friction at the cart can be modeled by c_0. The payload is m_1. The pendulum dynamics are modeled by c and k.

Flexible joint robot manipulator can also be modeled as two-mass rigid-flexible system. The motor hub's inertia is m_0. The viscous friction at motor bearing can be modeled as c_0. The payload is m_1. The flexible joint is modeled as c and k.

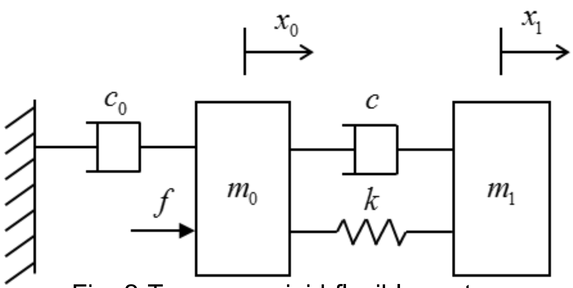

Fig. 2 Two-mass rigid-flexible system.

The equations of motion of the system in Fig. 2 can be found as

$$m_0\ddot{x}_0 + c\left(\dot{x}_0 - \dot{x}_1\right) + k\left(x_0 - x_1\right) + c_0\dot{x}_0 = f,$$
$$m_1\ddot{x}_1 - c\left(\dot{x}_0 - \dot{x}_1\right) - k\left(x_0 - x_1\right) = 0.$$

The corresponding transfer function from x_0 to x_1 is given by

$$P_2(s) = \frac{X_1(s)}{X_0(s)} = \frac{cs+k}{m_1 s^2 + cs + k}, \quad (1)$$

and from f to x_0 is given by

$$P_1(s) = \frac{X_0(s)}{F(s)} = \frac{m_1 s^2 + cs + k}{\left[\begin{array}{l} m_0 m_1 s^4 + \left(m_0 c + m_1 c + m_1 c_0\right)s^3 \\ +\left(m_0 k + m_1 k + c c_0\right)s^2 + c_0 k s \end{array}\right]}. \quad (2)$$

In most rigid-flexible systems, such as cranes, the command input is velocity instead of acceleration or force. Therefore, from (1) and (2), the transfer function from the velocity command v to x_1 is given by

$$P(s) = \frac{X_1(s)}{V(s)} = \frac{cs+k}{\left[\begin{array}{l} m_0 m_1 s^3 + \left(m_0 c + m_1 c + m_1 c_0\right)s^2 \\ +\left(m_0 k + m_1 k + c c_0\right)s + c_0 k \end{array}\right]}. \quad (3)$$

For simulation purpose, let $m_0 = 2\,\text{kg}$, $m_1 = 3\,\text{kg}$, $c = 0.1\,\text{kg.s}^{-1}$, $c_0 = 30\,\text{kg.s}^{-1}$, and $k = 1\,\text{kg.s}^{-2}$. The response x_1 from a unit-step velocity input v is shown in Fig. 3, where the effect of the flexible mode, with $\omega_n = 0.58\,\text{rad.s}^{-1}$ and $\zeta = 5.8\times10^{-2}$, is evident.

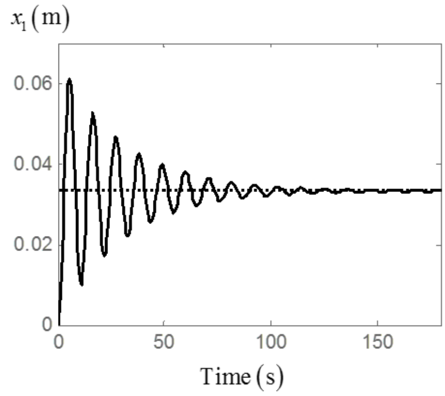

Fig. 3 Unit-step velocity response of the two-mass rigid-flexible system.

3. Performance Degradation from Acceleration Limit

This section shows that the performance of input shaping is degraded by acceleration limit, which is present in every practical actuators.

3.1 ZVD input shaping

Input shaping is based on destructive interference of impulse responses. A good tutorial paper is [3].

The ratio between the n-impulse response amplitude at time $t \ge t_n$ and the single-impulse response amplitude at time $t \ge t_1$ is given by

$$V(\omega_n,\zeta) = e^{-\zeta\omega_n t_n}\sqrt{\left[C(\omega_n,\zeta)\right]^2 + \left[S(\omega_n,\zeta)\right]^2},$$
$$C(\omega_n,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega_n t_i}\cos\left(\omega_n\sqrt{1-\zeta^2}\,t_i\right),$$
$$S(\omega_n,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega_n t_i}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t_i\right),$$

where V is the so-called *percentage vibration*, normally used in the literature to quantify the residual vibration, ω_n is the natural frequency of the applied linear system, ζ is its damping ratio, t_i is the time the i^{th} impulse is applied and A_i is the i^{th} impulse's amplitude.

The amplitudes A_i and time locations t_i of the impulse sequence are computed by solving the following equations:

$$V(\omega_n, \zeta) = 0, \tag{4}$$

$$\frac{\partial V(\omega_n, \zeta)}{\partial \omega_n} = 0, \tag{5}$$

$$\sum_{i=1}^{n} A_i = 1, \tag{6}$$

$$t_1 = 0, \tag{7}$$

which requires the knowledge of ω_n and ζ.

Eqs. (4) - (7) are used to solve six unknowns, which are the amplitudes and time locations of three impulses:

$$t_1 = 0,\ A_1 = \frac{1}{1+2K+K^2}, \tag{8}$$

$$t_2 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}},\ A_2 = \frac{2K}{1+2K+K^2}, \tag{9}$$

$$t_3 = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}},\ A_3 = \frac{K^2}{1+2K+K^2}, \tag{10}$$

$$K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}. \tag{11}$$

This three-impulse input shaper is known in the literature as zero-derivative-vibration (ZVD) shaper. It was proposed by [4].

3.2 Performance degradation from acceleration limit

The simulation result in Fig. 3 is redone. However, this time the unit-step velocity command is shaped by the ZVD input shaper, given by (8) - (11). Fig. 4(Top) contains the response x_1 with and without input shaper. Fig. 4(Bottom) shows the shaped unit-step velocity command v_s. It can be seen that, with input shaper, the response x_1 does not vibrate and the settling time is improved substantially.

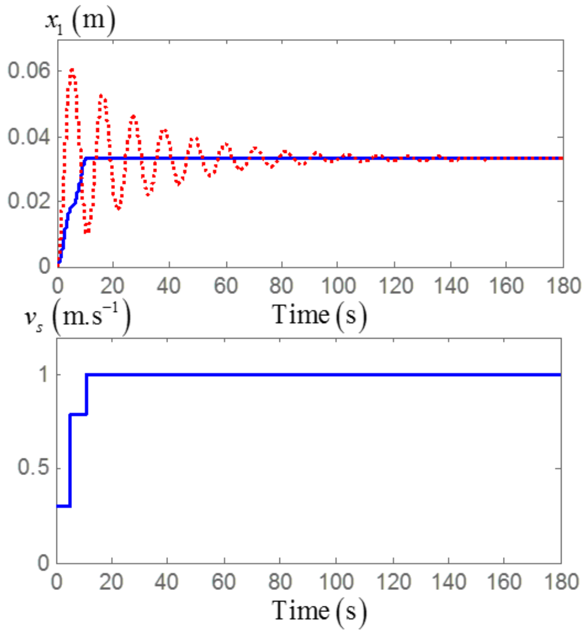

Fig. 4 (Top) Unit-step velocity response of the two-mass rigid-flexible system. Solid line is shaped response. Dash line is unshaped response. (Bottom) Shaped unit-step velocity command.

However, most actuators in practice have acceleration limit. Fig. 5 contains a diagram showing input shaping under acceleration limit. v_b is the original baseline velocity command, which is a step function of a magnitude v_f. v_s is the shaped velocity command, a staircase function. v_a is the acceleration-limited velocity command.

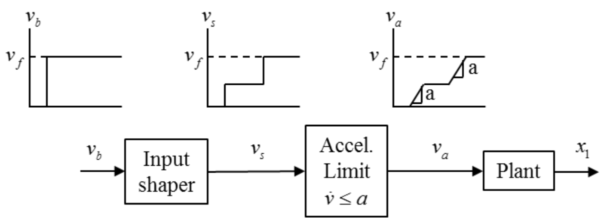

Fig. 5 Input shaping under acceleration limit.

v_s is designed to suppress residual vibration of the plant output x_1. However, because of the acceleration limit $\dot{v} \le a$, where a is a constant, v_a is given to the plant instead of v_s. As a result, the vibration suppression performance of the input shaper is degraded.

Fig. 6(Top) compares between the plant output x_1 with and without acceleration limit. When an acceleration limit $\dot{v} \le 0.1\,\mathrm{m.s}^{-2}$ exists, the vibration suppression performance of the input shaper degrades as can be seen from increasing oscillation in x_1 from v_a. Fig. 6(Bottom) provides the baseline command v_b, the shaped command v_s, and the acceleration-limited command v_a. It can be seen that the slope of v_a is limited to $0.1\,\mathrm{m.s}^{-2}$.

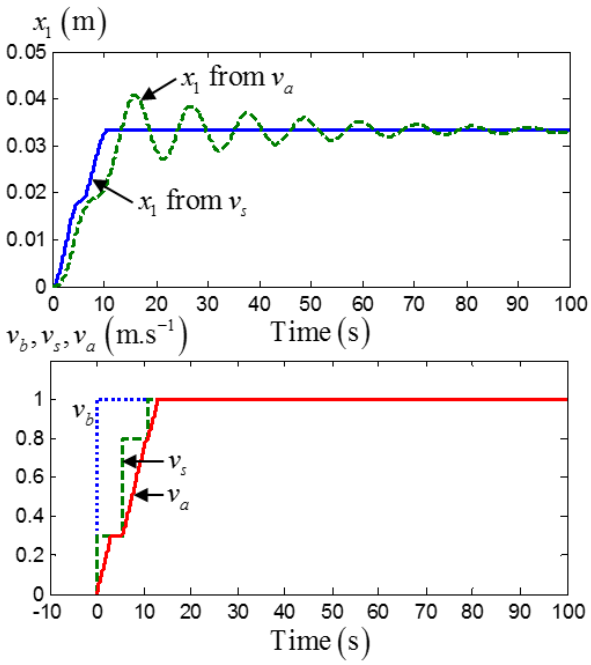

Fig. 6 (Top) Unit-step velocity response of the two-mass rigid-flexible system. Solid line is shaped response. Dash line is response under acceleration limit. (Bottom) Unit-step baseline velocity command v_b. Shaped command v_s. Acceleration-limited command v_a.

4. Baseline Velocity Command That Avoids Violating the Acceleration Limit

In this section, a new baseline velocity command that avoids violating the acceleration limit is found in closed form. This baseline command v_b will produce the shaped command v_s whose slopes will not exceed the acceleration limit $\dot{v} \le a$. Therefore, v_a will be exactly the same as v_s, and the performance of the input shaper will not be degraded.

Consider a case when the input shaper has two impulses, $A_1 > A_2$. Suppose the baseline command v_b is modified as a ramp-plus-step function as shown in Fig. 7. The ramp slope is given by a_1 / Δ, where a_1 is the desired final velocity and Δ is the ramp rise time to be designed.

Fig. 7 also shows the convolution result between v_b and the input shaper's impulse sequence. The shaped command v_s will have two ramp steps. The ramp rise time Δ is designed to ensure that

$$a = \frac{a_1 A_1}{\Delta} > \frac{a_1 A_2}{\Delta},$$

so the slope of v_s will not exceed the acceleration limit a. This procedure in modifying the baseline command can easily be extended to input shapers having more impulses or negative impulses.

Note that relationships

$$t_{i+1} - t_i \ge \frac{a_1 |A_i|}{a}, \quad \forall i = 1, 2, \ldots, n-1,$$

where n is the number of impulses, must be enforced to ensure that the rise time of each step in v_s is always less than the time between the step changes in v_b.

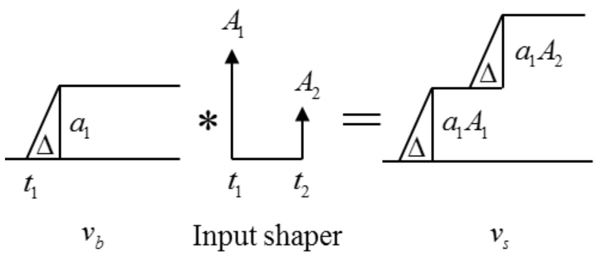

Fig. 7 Convolution between modified baseline command and input shaper's impulse sequence.

In our simulation example, the ZVD input shaper, given by (8) - (11), has $A_1 = 0.2973$, $A_2 = 0.4959$, $A_3 = 0.2068$, $t_1 = 0$, $t_2 = 5.4538$, and $t_3 = 10.9076$.

With an acceleration limit $\dot{v} \le a = 0.1\,\mathrm{m.s^{-2}}$ and $a_1 = 1\,\mathrm{m.s^{-1}}$, the ramp rise time is computed as $\Delta = a_1A_2 / a = 4.959\,\mathrm{s}$.

Fig. 8(Bottom) shows the modified baseline command v_b as well as the corresponding shaped command v_s and acceleration-limited command v_a. It can be seen that v_b has been modified such that v_s will not violate the acceleration limit; therefore, $v_s = v_a$. The input shaper performance will not be degraded because the shaped command is not distorted by the acceleration limit. Fig. 8(Top) contains the original, unmodified baseline command v_b along with its corresponding v_s and v_a for comparison.

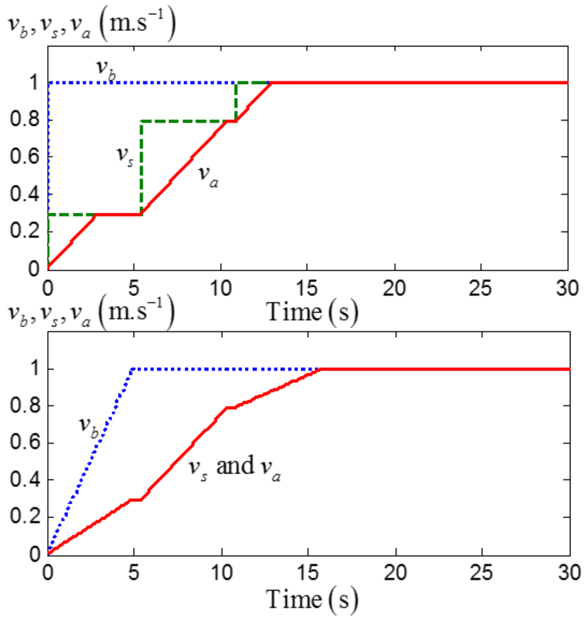

Fig. 8 Unit-step baseline velocity command v_b. Shaped command v_s. Acceleration-limited command v_a. (Top) Original baseline command. (Bottom) Modified baseline command.

Fig. 9 shows the system output x_1 when the original baseline velocity command v_b is used and when the modified baseline velocity command v_b is used. It can be seen that, with modified baseline command, the residual vibration is suppressed even under acceleration limit. This is because the modified command v_b is designed such that the modified shaped command v_s is not distorted by the acceleration limit.

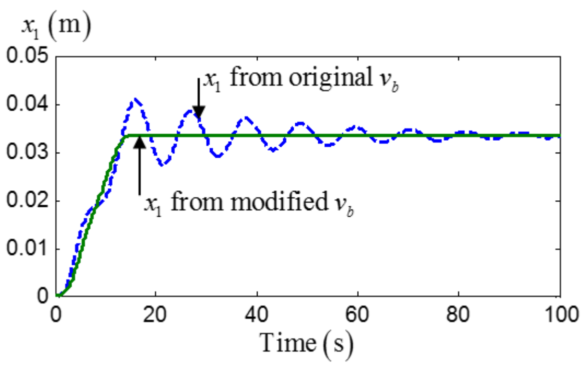

Fig. 9 Unit-step velocity response of the two-mass rigid-flexible system. Solid line is the result from

modified v_b. Dash line is response from the original, unmodified v_b.

More steps can be added to the modified baseline command. Fig. 10 shows a modified baseline command v_b with two steps. The two steps are placed at the impulse time locations t_1 and t_2. Fig. 10 also contains the convolution between the modified baseline command and the input shaper's impulse sequence.

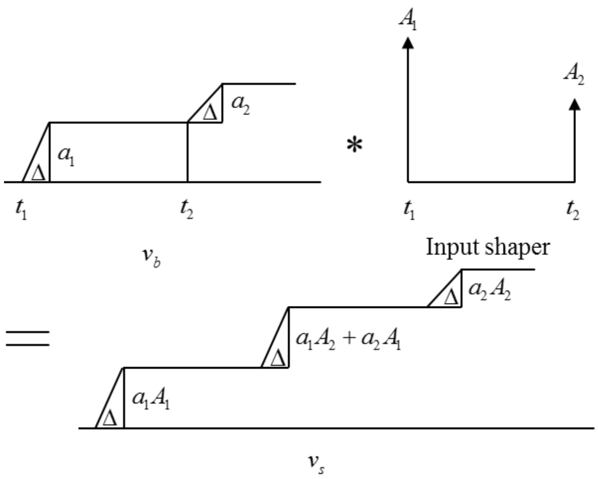

Fig. 10 Convolution between modified baseline command with two steps and input shaper's impulse sequence.

There are three unknowns: a_1, a_2, and Δ. They can be solved from relationships:

$$a_1 + a_2 = V,$$

$$\frac{a_1A_1}{\Delta} = \frac{a_1A_2 + a_2A_1}{\Delta} = a,$$

where V is the desired final velocity and a is the acceleration limit.

Modified baseline command with additional steps provides more robustness to parameter uncertainty.

5. Conclusions

Acceleration limit exists in most actuators in practice. The shaped command can be distorted by the acceleration limit resulting in degradation in vibration reduction performance.

For the first time, this paper proposes a modification to the baseline command so that the shaped command will not violate the acceleration limit.

It is shown by simulation with a rigid-flexible model that, under acceleration limit, the modified baseline command is not affected by the acceleration limit and can produce response without vibration.

Future work includes applying the proposed technique to actual hardware.

6. References

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