DRC-25

Model Reference Input Shaping Using State-Feedback Backstepping Controller

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Abstract

Input shaping is a technique to reduce residual vibration by destructive interference of impulse responses, that is, an impulse response can be cancelled by another impulse response, given appropriate impulse amplitudes and applied times. The input shaper is designed using the system mode parameters, which are natural frequencies and damping ratios. Therefore, its performance is affected by the inaccuracy of these system parameters. In this paper, for the first time, a state-feedback backstepping controller is applied to a flexible system to match the closed-loop system with a known reference model. The input shaper is then designed using the reference model's parameters, which are accurate. The backstepping controller can handle unmatched uncertainty. The advantages of the proposed technique are as follows: the flexible system is allowed to be time-varying or nonlinear, the input shaper can have short duration because it does not have to be robust, vibration induced by uncertainty can be reduced. Simulation results confirm the effectiveness of the proposed technique.

***Keywords**:* Input shaping; Vibration reduction; Backstepping; Model reference.

1. Introduction

Input shaping technique is used to reduce residual vibration, which is the vibration after the system reaches its destination, from point-to-point movement of flexible systems. Input shaping was originated by [1] and was made robust by [2]. Tutorial materials are available in [3].

Input shaping is designed from the knowledge of the mode parameters (natural frequency and damping ratio) of the flexible system. Therefore, its performance depends on the accuracy of this knowledge. When the plant model is not accurate or uncertain, the performance of the input shaper degrades.

Several works in the past were proposed to solve this problem. They include robust input shaping [4], [5], and [6] and adaptive input shaping [7], [8], [9], and [10]. Robust input shapers add more impulses into the impulse sequence representing the input shaper. Additional impulses results in longer settling time. Adaptive input shaping, where the input shapers adapt to the changing mode parameters, are usually complicated, can cover small range of parameter uncertainty, and may not be guaranteed to converge.

Another way to solve the problem is by using feedback control together with input shaping. The feedback control is designed to match the closed-loop system to a reference model, so that the input shaper can be designed from mode parameters of the reference model which are known precisely.

Ref. [11] proposed an input shaping method that increases the robustness of the design while does not lengthen the shaper duration. An adaptive control scheme was proposed to adapt a feedback controller on-line so that the closed-loop system matches a fixed reference model. The input shaper is then designed according to the natural frequency and damping ratio of the reference model. However, this work is limited to a second-order plant and a simple lead/lag controller.

Ref. [12] presented a model reference input shaping scheme using discrete-time sliding mode control for model matching. The sliding mode control has two phases: reaching and sliding phases. An intelligent system was used during the reaching phase.

In this paper, for the first time, backstepping control [13] is applied to match the closed-loop system to a reference model. The backstepping control breaks the design for the whole system into the designs for the subsystems. It therefore can handle unmatched uncertainties and disturbances naturally. In this work the plant parameters, which are damping and spring constants, are assumed to be ten-time uncertainty. Moreover, the uncertainty can exist in every part of the system. It was shown through simulation that the proposed control system can suppress the residual vibration of the flexible system effectively even under large amount of model uncertainty.

This paper is organized in this way. Section 2 contains model of a two-mass rigid-flexible system to be used in simulations. Section 3 presents the proposed technique, which is the model reference input shaping using state-feedback backstepping control. It contains backstepping control, using backstepping in model matching, and zero-vibration (ZV) input shaping. Section 4 discusses simulation results comparing open-loop input shaping to the proposed technique. Section 5 are conclusions.

2. Two-Mass Rigid-Flexible System

In this work, a two-mass rigid-flexible system in Fig. 1 is considered. In general, the system represents two entities, connected via a flexible part, which encompasses a large majority of actual rigid-flexible systems. The driving one has an absolute position and mass of s_0 and m_0, and the driven one has s_1 and m_1. k, c_0, and c are spring stiffness and two damping constants. f is the control force. The objective is to move both masses from the origin to a displacement X with zero residual vibrations and in a shortest time possible T.

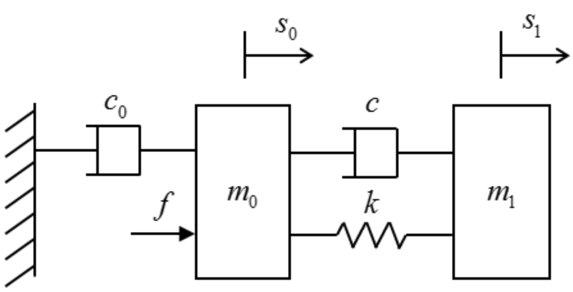

Fig. 1 Two-mass rigid-flexible system.

The equations of motion of the system in Fig. 1 can be found as

$$m_0\ddot{s}_0 + c(\dot{s}_0 - \dot{s}_1) + k(s_0 - s_1) + c_0\dot{s}_0 = f,$$
$$m_1\ddot{s}_1 - c(\dot{s}_0 - \dot{s}_1) - k(s_0 - s_1) = 0.$$

In most rigid-flexible systems, such as cranes, the command input is velocity v instead of acceleration or force f.

The corresponding state-space model is then given by

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k}{m_1} & -\dfrac{c}{m_1} & \dfrac{k}{m_1} & \dfrac{c}{m_1} \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{m_0} & \dfrac{c}{m_0} & -\dfrac{k}{m_0} & -\dfrac{(c+c_0)}{m_0} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}$$

$$+\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{m_0} \end{Bmatrix} f,\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \qquad (1)$$

where $x_1 = s_1$, $x_2 = \dot{s}_1$, $x_3 = s_0$, $x_4 = \dot{s}_0$, and $f = \dot{v}$.

For simulation purpose, let $m_0 = 2\,\text{kg}$, $m_1 = 3\,\text{kg}$, $c = 0.1\,\text{kg.s}^{-1}$, $c_0 = 30\,\text{kg.s}^{-1}$, and $k = 1\,\text{kg.s}^{-2}$. The response x_1 from a unit-step velocity input v is shown in Fig. 2, where the effect of the flexible mode, with $\omega_n = 0.58\,\text{rad.s}^{-1}$ and $\zeta = 5.8\times10^{-2}$, is evident.

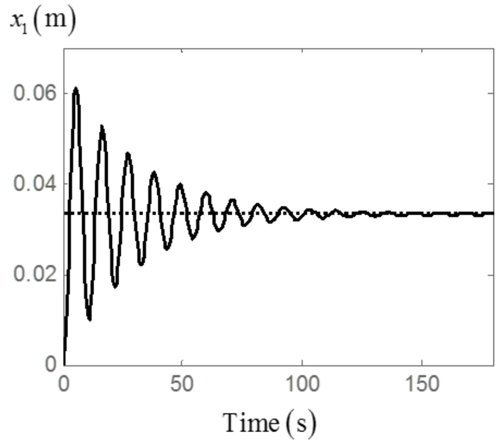

Fig. 2 Impulse response of the two-mass rigid-flexible system.

3. Model Reference Input Shaping Using State-Feedback Backstepping Control

This section discusses the use of a simple state-feedback backstepping control for model matching.

3.1 State-feedback backstepping control

The flexible plant model (1) can be written in the so-called strict-feedback form

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2, \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3, \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)x_4, \\ \dot{x}_4 &= f_4(x_1, x_2, x_3, x_4) + g_4(x_1, x_2, x_3, x_4)f, \end{aligned} \qquad (2)$$

where f_i and g_i are functions of states. Note that, to put (1) in the strict-feedback form, the term $(c/m_1)x_4$ is neglected. It will be treated as plant uncertainty, which will be handle by the control system.

State-feedback backstepping controller for the system (2) can be described by a diagram in Fig. 3. x_{1d} is a reference input. With the system output $y = x_1$, a virtual control x_{2d} that achieves tracking of x_1 to x_{1d} is computed. With the system state x_2, a virtual control x_{3d} that achieves tracking of x_2 to x_{2d} is computed.

Similarly, x_{4d} and f are computed. f is the actual control given to the plant.

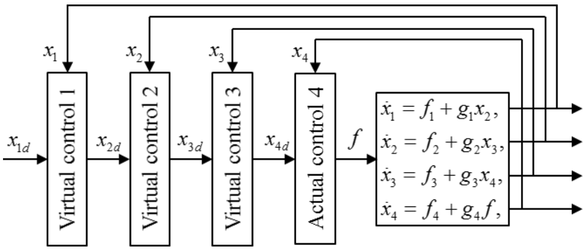

Fig. 3 State-feedback backstepping control.

Because the backstepping control creates a virtual control for every subsystem $\dot{x}_i = f_i + g_i x_{i+1}$, it can handle model uncertainty and disturbance in the subsystem, that is, it can effectively handle "unmatched" uncertainty and disturbance.

3.2 Model matching

The proposed system is shown in Fig. 4. The dash box contains closed-loop system. The backstepping control is aimed to reduce the difference between the plant output y and the reference model output y_m. As a result, the closed-loop system, which is a mapping from the shaped velocity command v to y will be close to the known reference model, which is a mapping from v to y_m. The input shaper then can be designed from mode parameters of the reference model, which are exactly known.

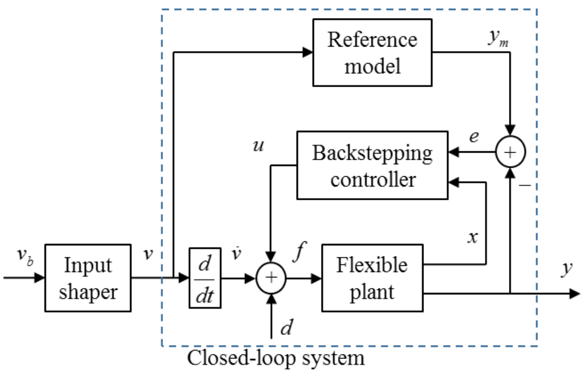

Fig. 4 Proposed model reference input shaping using state-feedback backstepping control.

First, consider the first subsystem in (2), which is $\dot{x}_1 = f_1 + g_1 x_2$. Let errors be $e_1 = y - y_m$ and $e_i = x_i - x_{id}, \forall i = 2, 3, 4$. The error dynamic of the first subsystem is then given by

$$\begin{aligned}\dot{e}_1 &= \dot{x}_1 - \dot{y}_m \\ &= f_1 + g_1 x_2 - \dot{y}_m \\ &= f_1 + g_1 e_2 + g_1 x_{2d} - \dot{y}_m.\end{aligned}$$

Choose a Lyapunov function $V_1 = (1/2)e_1^2$. Its derivative is given by

$$\begin{aligned}\dot{V}_1 &= e_1 \dot{e}_1 \\ &= e_1 \left(f_1 + g_1 e_2 + g_1 x_{2d} - \dot{y}_m \right).\end{aligned} \tag{3}$$

Letting the virtual control x_{2d} be

$$x_{2d} = g_1^{-1}\left(-f_1 + \dot{y}_m - c_1 e_1\right),\ c_1 > 0, \tag{4}$$

(3) becomes

$$\dot{V}_1 = -c_1 e_1^2 + g_1 e_1 e_2.$$

Similarly, for the second subsystem in (2), which is $\dot{x}_2 = f_2 + g_2 x_3$, the error dynamic is given by

$$\dot{e}_2 = f_2 + g_2 e_3 + g_2 x_{3d} - \dot{x}_{2d}.$$

The derivative of a Lyapunov function $V_2 = V_1 + (1/2)e_2^2$ is given by

$$\dot{V}_2 = \dot{V}_1 + e_2 \left(f_2 + g_2 e_3 + g_2 x_{3d} - \dot{x}_{2d} \right). \tag{5}$$

Letting the virtual control x_{3d} be

$$x_{3d} = g_2^{-1}\left(-f_2 + \dot{x}_{2d} - c_2 e_2 - g_1 e_1\right),\ c_2 > 0, \tag{6}$$

(5) becomes

$$\dot{V}_2 = -c_1 e_1^2 - c_2 e_2^2 + g_2 e_2 e_3. \tag{7}$$

For the third subsystem in (2), which is $\dot{x}_3 = f_3 + g_3 x_4$, the virtual control x_{4d} and the derivative of the Lyapunov function $V_3 = V_2 + (1/2)e_3^2$ become

$$x_{4d} = g_3^{-1}\left(-f_3 + \dot{x}_{3d} - c_3 e_3 - g_2 e_2\right),\ c_3 > 0, \quad (8)$$

$$\dot{V}_3 = -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 + g_3 e_3 e_4.$$

Finally, for the last subsystem, which is $\dot{x}_4 = f_4 + g_4 f = f_4 + g_4 u + g_4 \dot{v}$, the actual control u and the derivative of the Lyapunov function $V_4 = V_3 + (1/2)e_4^2$ become

$$u = g_4^{-1}\left(-f_4 + \dot{x}_{4d} - c_4 e_4 - g_3 e_3 - g_4 \dot{v}\right), \quad c_4 > 0, \quad (9)$$

$$\dot{V}_4 = -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 - c_4 e_4^2 < 0.$$

From the Lyapunov's stability theorem, all errors will go to zero asymptotically.

3.3 ZV input shaping

Input shaping is based on destructive interference of impulse responses. The ratio between the n-impulse response amplitude at time $t \geq t_n$ and the single-impulse response amplitude at time $t \geq t_1$ is given by

$$V(\omega_n,\zeta) = e^{-\zeta\omega_n t_n}\sqrt{\left[C(\omega_n,\zeta)\right]^2 + \left[S(\omega_n,\zeta)\right]^2},$$

$$C(\omega_n,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega_n t_i}\cos\left(\omega_n\sqrt{1-\zeta^2}\,t_i\right),$$

$$S(\omega_n,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega_n t_i}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t_i\right),$$

where V is the so-called *percentage vibration*, normally used in the literature to quantify the residual vibration, ω_n is the natural frequency of the applied linear system, ζ is its damping ratio, t_i is the time the i^{th} impulse is applied and A_i is the i^{th} impulse's amplitude.

The amplitudes A_i and time locations t_i of the impulse sequence are computed by solving the following equations:

$$V(\omega_n,\zeta) = 0, \quad (10)$$

$$\sum_{i=1}^{n} A_i = 1, \quad (11)$$

$$t_1 = 0, \quad (12)$$

which requires the knowledge of ω_n and ζ.

Eqns. (10) - (12) are used to solve four unknowns, which are the amplitudes and time locations of two impulses:

$$t_1 = 0,\ A_1 = \frac{1}{1+K}, \quad (13)$$

$$t_2 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}},\ A_2 = \frac{K}{1+K}, \quad (14)$$

$$K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}. \quad (15)$$

This two-impulse input shaper is known in the literature as zero-vibration (ZV) shaper.

4. Simulation Results

4.1 Open-loop input shaping

Consider first the open-loop input shaping as shown in Fig. 5. The flexible system is given by (1) with nominal parameter values. The ZV input shaper is given by (13) – (15). The natural frequency and damping ratio are computed from using nominal parameter values.

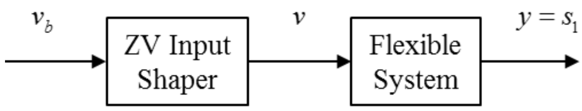

Fig. 5 Open-loop input shaping.

The baseline velocity command v_b is a square wave of magnitude one as shown in Fig. 6(Top). The resulting shaped velocity command v is a stair-case command as shown in Fig. 6(Bottom).

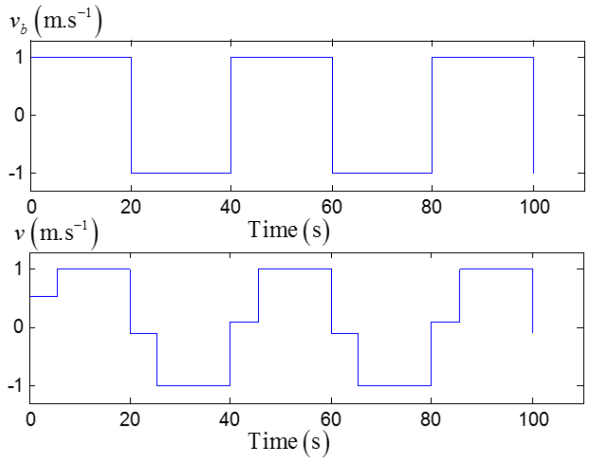

Fig. 6 (Top) Baseline velocity command. (Bottom) Shaped velocity command.

The resulting flexible plant output $y = s_1$ is shown in Fig. 7(Top). It can be seen that, when the plant model is perfect, using the ZV input shaper in open-loop perfectly cancels the residual vibration in the output.

However, when the plant model is not perfect. As an example, a number 1.5 is deliberately multiplied to all the damping ratios and the spring constant. The ZV input shaper is unchanged. The output when the model is uncertain is shown in Fig. 7(Bottom). It is obvious that there is substantial residual vibration in the plant output.

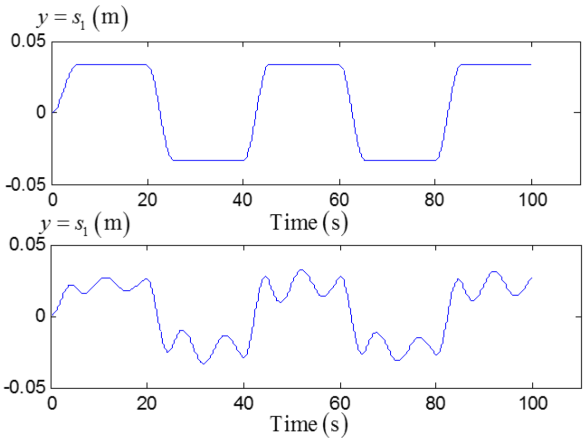

Fig. 7 (Top) Plant output when the plant model is perfect. (Bottom) Plant output when the model is uncertain.

4.2 Model reference input shaping using state-feedback backstepping control

In this section, the proposed control system in Fig. 4 is applied. The backstepping controller matches the closed-loop system from v to y to the reference model from v to y_m. As a result, the input shaper, which is designed from mode parameters of the reference model, is almost unaffected by the uncertainty in the flexible plant. Besides, the flexible plant can be time-varying or nonlinear.

From the flexible plant (1) and the strict-feedback form (2), f_i and g_i are given by

$$f_1 = f_3 = 0,\ g_1 = g_3 = 1,$$

$$f_2 = -\frac{k}{m_1}x_1 - \frac{c}{m_1}x_2,\ \ g_2 = \frac{k}{m_1},$$

$$f_4 = \frac{k}{m_0}x_1 + \frac{c}{m_0}x_2 - \frac{k}{m_0}x_3 - \frac{(c+c_0)}{m_0}x_4,$$

$$g_4 = \frac{1}{m_0}.$$

Choose the controller constants $c_i = 1$, $\forall i = 1, 2, 3, 4$. The state-feedback backstepping control law is given by (9) with virtual control laws (4), (6), and (8). The reference model is the plant (1) with nominal parameter values. A simulation was written using a sampling period of 5 ms.

Fig. 8 shows the states and their desired values when a number 10 is deliberately multiplied to all the damping ratios and the spring constant to simulate uncertainty in the parameter values. The desired values are in dash lines whereas the states are in solid lines.

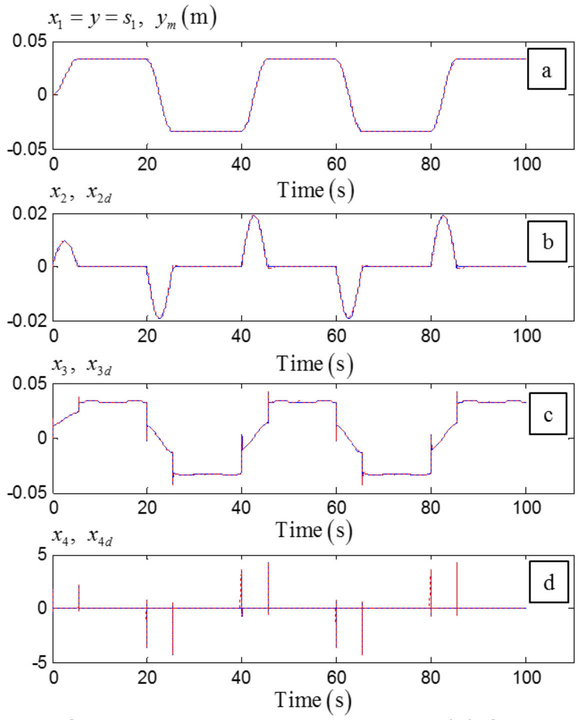

Fig. 8 States and their desired values. (a) System output $x_1 = y = s_1$ and its desired value y_m. (b) State x_2 and its desired virtual control x_{2d}. (c) State x_3 and its desired virtual control x_{3d}. (d) State x_4 and its desired virtual control x_{4d}.

It can be seen from Fig. 8 that all states are bounded and can follow their desired values closely even when large parameter uncertainties are present.

5. Conclusions

A model reference input shaping technique is presented. State-feedback backstepping control is used to match the closed-loop system to a reference model. As a result, the input shaper can be designed from mode parameters of the reference model, unaffected by the uncertainty in the plant parameters. The proposed input shaping system is shown via simulation on a practical flexible system that it can maintain good input shaping performance even when the plant uncertainty is substantially large.

Future work includes rejecting vibrations from external disturbances by sliding mode controller, using intelligent systems to identify plant if it is unknown, using observer to estimate unmeasured states, getting rid of the derivative terms in the control law, and implementing the proposed technique with actual flexible systems.

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DRC-25

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