Proceedings of the ASME 2017 International Mechanical Engineering Congress and Exposition
IMECE2017
November 3-9, 2017, Tampa, Florida, USA

IMECE2017-71364

INPUT SHAPER SYNTHESES FOR EXPLICIT FRACTIONAL DERIVATIVE SYSTEMS USING NONLINEAR OPTIMIZATION

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ABSTRACT

Fractional derivative system has gained its popularity in modeling and control because of its long memory property. Only recently, residual vibration suppression for the fractional derivative oscillatory system using input shaping has been studied. Input shaping suppresses residual vibration by using destructive interference of impulse responses. So far, only a few types of input shapers have been proposed for the fractional derivative oscillatory system, using only analytical, closed-form solutions. In this paper, input shaper syntheses for explicit fractional derivative systems using nonlinear optimization have been proposed. This work designs input shapers that have never been used with the fractional derivative system before, which include fixed-interval input shaper and specified-insensitivity input shaper, and extends the type that has already been used to a more general, improved input shaper.

INTRODUCTION

Fractional-order systems are based on differentiation and integration whose orders are not necessarily integer. It has even been shown that the real objects are generally fractional [1]. Even though fractional-order systems have been studied for more than three centuries, they have just become more useful in various science and engineering areas only recently. This is largely due to the rapid development of computer technology that helps in the realization and approximation of fractional derivative and integral.

Analogous to the integer-order differential equation, the fractional-order differential equation can have oscillatory response. The response of the *relaxation-oscillation* differential equation ([3] and [4]) becomes an underdamped oscillation when its order is between one and two and becomes an undamped oscillation when its order is two.

Posicast control was proposed by Smith [5] to suppress the oscillatory response of the integer-order differential equation. It is based on the cancellation of the impulse responses of the system, resulting in zero vibration. The impulse amplitudes as well as the apply time locations of the impulses are the design parameters to be found. Singer and Seering [6] added robustness to uncertainty to the technique and called it *input shaping*.

Limited amount of work has been done in designing the input shapers for the fractional-order systems. Poty et al. [10] designed the zero vibration (ZV) and the zero vibration and derivative (ZVD) input shapers for the relaxation-oscillation differential system. They considered only the oscillatory part of the response in order to obtain closed-form formulas for the impulse amplitudes and time locations. Poty et al. [7] considered both non-oscillatory and oscillatory parts of the response to obtain approximated formulas for the ZV and ZVD input shapers. The approximated formulas were designed by finding zeros of a series, representing the unit impulse and unit step responses. Abid et al. [8] designed the unity-magnitude (UM) input shaper for undamped and damped fractional-order derivative systems. They obtained closed-form formulas of the

UM input shaper for the undamped case, and approximated formulas for the damped case.

In this paper, for the first time, the problem of finding the impulse amplitudes and time locations of the input shapers for the fractional-order derivative systems is formulated as a nonlinear optimization problem. Four types of the input shapers were designed, namely the ZVD^k input shaper, the fixed interval input shaper, the unity-magnitude input shaper, and the specified-insensitivity input shaper.

PRELIMINARY

A. Second-Order System

The unit impulse response of a second-order, underdamped system is given by [2]

$$y(t)=\frac{1}{m\omega_d}e^{-\zeta\omega_n t}\sin\left(\omega_d t\right), \tag{1}$$

where m is the mass, ω_n is the natural frequency, ζ is the damping ratio, and $\omega_d=\omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency.

Having N impulses with amplitudes $A_i,\, i=1,2,\ldots,N,$ applied at time locations $t_i,$ the response at time $t\geq t_N$ is given by

$$\begin{aligned} y(t)&=\sum_{i=1}^{N}\left\{\frac{A_i}{m\omega_d}e^{-\zeta\omega_n(t-t_i)}\sin\left[\omega_d\left(t-t_i\right)\right]\right\} \\ &=A\sin\left(\omega_d t-\psi\right), \end{aligned} \tag{2}$$

where

$$A=\sqrt{\begin{aligned}&\left[\sum_{i=1}^{N}\frac{A_i}{m\omega_d}e^{-\zeta\omega_n(t-t_i)}\cos\left(-\omega_d t_i\right)\right]^2\\ &+\left[\sum_{i=1}^{N}\frac{A_i}{m\omega_d}e^{-\zeta\omega_n(t-t_i)}\sin\left(-\omega_d t_i\right)\right]^2\end{aligned}}$$

and

$$\psi=-\tan^{-1}\left[\frac{\sum_{i=1}^{N}\frac{A_i}{m\omega_d}e^{-\zeta\omega_n(t-t_i)}\cos\left(-\omega_d t_i\right)}{\sum_{i=1}^{N}\frac{A_i}{m\omega_d}e^{-\zeta\omega_n(t-t_i)}\sin\left(-\omega_d t_i\right)}\right].$$

Using (2), the response amplitude at $t=t_1$ with $A_1=1$ is given by

$$A_{\uparrow}=\frac{1}{m\omega_d}.$$

Using (2), the response amplitude at $t=t_N$ with $A_i,\, i=1,2,\ldots,N,$ is given by

$$A_{\Sigma}=\frac{1}{m\omega_d}e^{-\zeta\omega_n t_N}\sqrt{\left[C\left(\omega_n,\zeta\right)\right]^2+\left[S\left(\omega_n,\zeta\right)\right]^2},$$

where

$$C\left(\omega_n,\zeta\right)=\sum_{i=1}^{N}A_i e^{\zeta\omega_n t_i}\cos\left(\omega_d t_i\right),$$

$$S\left(\omega_n,\zeta\right)=\sum_{i=1}^{N}A_i e^{\zeta\omega_n t_i}\sin\left(\omega_d t_i\right).$$

The ratio of A_{Σ} to $A_{\uparrow}$ is then

$$\frac{A_{\Sigma}}{A_{\uparrow}}\triangleq V\left(\omega_n,\zeta\right)=e^{-\zeta\omega_n t_N}\sqrt{\left[C\left(\omega_n,\zeta\right)\right]^2+\left[S\left(\omega_n,\zeta\right)\right]^2}. \tag{3}$$

This ratio is called *percentage vibration* and is the ratio of the residual vibration amplitude under an input shaper to the residual vibration amplitude of a unit impulse response without input shaper. It is a measure of residual vibratory level, used most often in the input shaping literature.

The percentage vibration can be simplified further as

$$\begin{aligned} V&=e^{-\zeta\omega t_N}\sqrt{\left[C\left(\omega,\zeta\right)\right]^2+\left[S\left(\omega,\zeta\right)\right]^2} \\ &=e^{-\zeta\omega t_N}\left|\sum_{i=1}^{N}A_i e^{\omega t_i\left(\zeta+j\sqrt{1-\zeta^2}\right)}\right| \\ &=e^{-\zeta\omega_n t_N(\omega/\omega_n)}\left|\sum_{i=1}^{N}A_i e^{-t_i s_p(\omega/\omega_n)}\right|, \end{aligned} \tag{4}$$

where ω is the actual natural frequency, ω_n is the model natural frequency, and $s_p=-\zeta\omega_n-j\omega_n\sqrt{1-\zeta^2}$ is one of the system's flexible poles. The *sensitivity curve* is a plot between ω/ω_n and V using the last equality of (4). The curve shows the robustness of an input shaper to uncertainty in the model natural frequency.

B. Fractional Derivative System

Consider the so-called *relaxation-oscillation equation* ([3] and [4]),

$$\tau^n\bar{D}^n y(t)+y(t)=u(t), \tag{5}$$

where $u(t)$ is the input, $y(t)$ is the output, $\bar{D}$ is the differential operator, and $n,\tau\in\mathbb{R}^+$. This equation is one of the simplest fractional-order differential equations, found in control problems. From [4], the system decays for $n\leq 1,$ becomes damped oscillation for $1<n<2,$ and becomes undamped oscillation for $n=2.$

Laplace transform of (5), with zero initial conditions, leads to a transfer function

$$\frac{Y(s)}{U(s)}=\frac{1}{1+(\tau s)^n}. \tag{6}$$

For the case of damped oscillation, where $1<n<2,$ the unit impulse response is given by [7]

$$y(t) = L^{-1}\left[\frac{1}{1+(\tau s)^n}\right]$$
$$= \left[\frac{\tau^n \sin n\pi}{\pi}\int_0^\infty \frac{x^n e^{-xt}dx}{1+2(\tau x)^n \cos n\pi + (\tau x)^{2n}}\right] \quad (7)$$
$$-\left[\frac{2}{n}\tau^{-1}e^{t\tau^{-1}\cos\frac{\pi}{n}}\cos\left(t\tau^{-1}\sin\frac{\pi}{n}+\frac{\pi}{n}\right)\right].$$

The first part of the unit impulse response is not oscillatory whereas the second part is.

Comparing the oscillatory part of (7) to (1), the damping ratio of the fractional derivative system is related to the fractional order, n, as [8]

$$\zeta(n) = -\cos\left(\frac{\pi}{n}\right) \quad (8)$$

and the natural frequency is

$$\omega_n = \frac{1}{\tau}. \quad (9)$$

The percentage vibration (3) remains the same as

$$V(\omega_n,\zeta) = e^{-\zeta\omega_n t_N}\sqrt{\left[C(\omega_n,\zeta)\right]^2+\left[S(\omega_n,\zeta)\right]^2}, \quad (10)$$

but with

$$C(\omega_n,\zeta) = \sum_{i=1}^{N} A_i e^{\zeta\omega_n t_i}\cos\left(\omega_d t_i + \frac{\pi}{n}\right),$$
$$S(\omega_n,\zeta) = \sum_{i=1}^{N} A_i e^{\zeta\omega_n t_i}\sin\left(\omega_d t_i + \frac{\pi}{n}\right),$$

where ζ is given by (8), ω_n is given by (9), and $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Note that the last equality of (4) remains unchanged, so does the sensitivity curve.

C. Matlab Nonlinear Optimization Command

A Matlab command, *fmincon*, solves an optimization problem having nonlinear objective function or nonlinear constraints, given by

Find: x

To minimize: $f(x)$

Subject to constraints:

$$c(x) \le 0,\ c_{eq}(x) = 0,\ Ax \le b,\ A_{eq}x = b_{eq},\ lb \le x \le ub,$$

where $f(x)$ is a nonlinear scalar-valued function; $c(x)$ and $c_{eq}(x)$ are nonlinear vector-valued functions; A and A_{eq} are constant matrices; b, b_{eq}, lb, ub are constant vectors; and x is a vector of decision variables.

The command *fmincon* has the syntax as follows:

$$x = \text{fmincon}\left(objfun, x_0, A, b, A_{eq}, b_{eq}, \ldots \right.$$
$$\left. lb, ub, nonlcon, options\right) \quad (11)$$

where *objfun* is a function containing the objective function $f(x)$, x_0 is the initial guess of x, *nonlcon* is a function containing the nonlinear constraints $c(x)$ and c_{eq}, and *options* are other Matlab options.

ZVD^K INPUT SHAPER

The ZV and ZVD input shapers, proposed in [10] by considering only the oscillatory part of (7), have impulse amplitudes and time locations as

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & \frac{\pi}{\omega_d} \end{bmatrix} \quad (12)$$

and

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+2K+K^2} & \frac{2K}{1+2K+K^2} & \frac{K^2}{1+2K+K^2} \\ 0 & \frac{\pi}{\omega_d} & \frac{2\pi}{\omega_d} \end{bmatrix}, \quad (13)$$

respectively, with

$$K = e^{\pi\frac{\cos(\pi/n)}{\sin(\pi/n)}}.$$

Note that these ZV and ZVD shapers are the same as those of [11] where ζ is given by (8), ω_n is given by (9), and $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Therefore, the general ZVD^k input shaper can be found from generalized formulas:

$$A_i = \frac{\binom{k+1}{i-1}K^{i-1}}{\sum_{j=0}^{k+1}\binom{k+1}{j}K^j},\ t_i = (i-1)\frac{\pi}{\omega_n\sqrt{1-\zeta^2}},$$
$$i = 1, 2, \ldots, k+2,$$

where

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

is the combination of n things taken r at a time.

Ref. [7] considered both non-oscillatory and oscillatory parts of (7). The ZV and ZVD input shapers were designed by finding zeros of the series, representing the unit impulse and unit step responses. The impulse amplitudes and time locations of the resulting input shapers were given by approximated formulas. For the ZV input shaper,

$$t_1 = 0,$$
$$t_2 = \frac{1}{\tau}\left[-22.9+16.38n+18.65(n-2)^2+13.88(n-2)^3\right.$$
$$+9.2(n-2)^4+4.36(n-2)^5+2.33(n-2)^6+0.62(n-2)^7$$
$$\left.+0.48(n-2)^8\right]^{1/n},$$

$$A_1 = \frac{1}{1+D},\; A_2 = \frac{D}{1+D},\; D = \frac{79.195n^2 - 138.507n + 59.528}{100}. \quad (14)$$

For the ZVD input shaper, t_1 and t_2 were the same as those of the ZV input shaper,

$$t_3 = t_2 + \frac{\pi}{\omega_d},$$

$$A_1 = \frac{1}{1+2D+D^2},\; A_2 = \frac{2D}{1+2D+D^2},\; A_3 = \frac{D^2}{1+2D+D^2}. \quad (15)$$

In this and following sections, the open-loop input shaping system as shown in Fig. 1 will be applied where u_s is the shaped input.

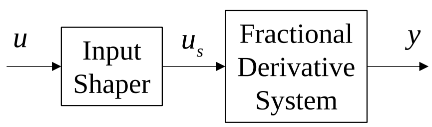

Fig. 1 Open-loop input shaping system.

Consider a fractional derivative system (6), with $n = 1.9$ and $\tau = 1$ as in [8]. The system has damped oscillation. Use a servo sampling time, $t_s = 0.01$ s. Table 1 contains the impulse amplitudes and time locations of the ZV and ZVD input shapers, according to (12)–(15). Note that, when the non-oscillatory part of (7) is taken into account in the design of the ZV and ZVD input shapers (Eqns. (14) and (15)), the first impulse amplitude is smaller and the input shaper length is shorter. This is a result of the overshoot reduction from the non-oscillatory part by the input shaper.

TABLE I. ZV AND ZVD INPUT SHAPERS

Input shaper	**Impulse amplitudes and time locations**
ZV in (12)	$A_1 = 0.565,\ A_2 = 0.435,$ $t_1 = 0,\ t_2 = 3.152.$
ZVD in (13)	$A_1 = 0.319,\ A_2 = 0.492,\ A_3 = 0.189,$ $t_1 = 0,\ t_2 = 3.152,\ t_3 = 6.304.$
ZV in (14)	$A_1 = 0.549,\ A_2 = 0.451,$ $t_1 = 0,\ t_2 = 3.064.$
ZVD in (15)	$A_1 = 0.301,\ A_2 = 0.495,\ A_3 = 0.204,$ $t_1 = 0,\ t_2 = 3.064,\ t_3 = 6.217.$

FIXED-INTERVAL INPUT SHAPER

In this section, an input shaper having a fixed time interval, ΔT, between impulses is obtained using optimization problem for the underdamped, fractional derivative system (6).

Because the time locations, $t_i, i = 1, 2, ..., N$, of the impulses are known, only the impulse amplitudes, $A_i, i = 1, 2, ..., N$, are to be found. The proposed optimization problem to find the impulse amplitudes is given as follows:

Find: $A_i, i = 1, 2, ..., N$

To minimize: V in (10)

Subject to constraints:

$$\sum_{i=1}^{N} A_i = 1, \quad (16)$$

$$A_i \geq 0,\ i = 1, 2, ..., N, \quad (17)$$

where (16) is to normalize the amplitudes so the final value of u_s equals that of u, and (17) is to obtain only positive impulses. This optimization problem can be solved using the *fmincon* command (11) with $A_{eq} = [1, 1, ..., 1]$, $b_{eq} = 1$, and $lb = [0, 0, ..., 0]^T$.

Consider a fractional derivative system (6), with $n = 1.9$ and $\tau = 1$ as in [8]. Using a servo sampling time, $t_s = 0.01$ s, the fixed time interval, $\Delta T = 0.1$ s, the initial guess, $x_0 = [1, 1, ..., 1]^T$, and the number of impulses, $N = 40$, the simulation result is shown in Fig. 2. The original input, u, is a unit step input. Fig. 2(a) contains the output, y, and the input, u. The dotted line and the solid line are the outputs without and with the input shaper, respectively. It can be seen that, without the input shaper, the output oscillates severely. Note that, with the input shaper, small overshoot is still present due to the non-oscillatory part of the unit impulse response (7). Fig. 2(b) shows the impulse amplitudes, A_i, versus their time locations, t_i. Fig. 2(c) presents the sensitivity curve. Note that at large values of ω / ω_n, the minimum value of the percentage vibration, V, is larger than zero due to the phase shift, π / n, in the objective function (10).

To simulate the fractional derivative system (6), the command *nid* of the N-integer Matlab toolbox [9] was used to approximate the fractional-order term, $(\tau s)^n$. The bandwidth was set equal to $[10^{-4}, 10^4]$, the number of zeros and poles of the approximating transfer function was 10, and the approximating formula was 'crone'.

Note that when the number of impulses, N, is reduced to $N = 31$, which is equivalent to the shaper duration of 3 seconds, the ZV input shaper in Table I is recovered by the fixed-interval input shaper, as shown in Fig. 2(d) with $A_1 = 0.562$, $A_{31} = 0.438$, and $A_i = 0, i = 2, 3, ..., 30$.

Using the optimization problem, the fixed-interval input shaper can be designed to be robust to plant model uncertainty. The objective function, V, in (10) is a function of ω_n and ζ, which are related to the plant parameters, n and τ, by (8) and (9). Hence, to find robust input shaper, the objective function of the optimization problem can be set equal to the average of the functions, V, evaluated at different values of ω_n or ζ. For example, consider the case when the plant parameter, τ, is uncertain and can have a value in a closed set, $\tau \in [0.5, 1.5]$, with a nominal value of $\tau = 1$. From (9), this is equivalent to the actual natural frequency being in a closed set, $\omega \in [0.67, 2]$, and the model natural frequency being $\omega_n = 1$ rad/s. Let the other plant parameter be $n = 1.9$. From (8), the damping ratio is $\zeta = 0.083$. Let the objective function be the average of $V(0.67, 0.083)$, $V(1, 0.083)$, $V(1.33, 0.083)$, and $V(2, 0.083)$. Fig. 3(a) shows the impulse amplitudes of the resulting fixed-interval input shaper with $N = 96$ impulses and an input shaper length of 9.50 seconds. Fig. 3(b) contains the sensitivity curve. Notice that the percentage vibration, *V*, is minimized for $\omega = 0.67$, 1.33, and 2 rad/s. Fig. 3(c) presents the shaped input, u_s, in the dashed line and the output, *y*, in the solid line when the plant parameter, τ, was set to $\tau = 1.5$, which is 50% higher than its nominal, model value. The designed input shaper can suppress the residual vibration in the output well even with 50% uncertainty. Fig. 3(d) shows the impulse amplitudes of the ZVDD input shaper with 4 impulses. The ZVDD input shaper length is 9.45 seconds, which is comparable to that of the designed fixed-interval input shaper. Fig. 3(e) contains the sensitivity curve, and Fig. 3(f) presents the output and the shaped input. Notice that the residual vibration still remains in the ZVDD case.

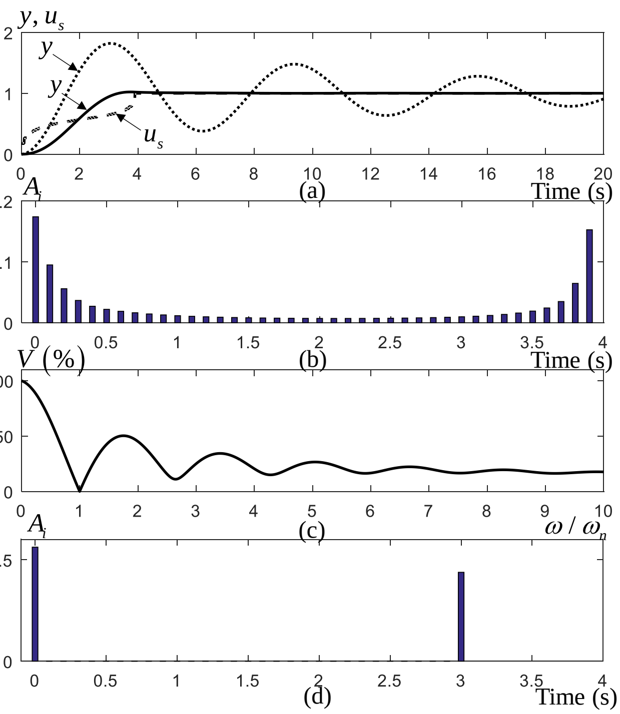

Fig. 2 (a) Dotted line and solid line are the outputs without and with the input shaper, respectively. Dashed line is the shaped input. (b) Input shaper amplitudes. (c) Sensitivity curve. (d) Input shaper amplitudes.

Fixed-interval input shaper, having negative impulses, can be obtained by not enforcing the lower bound, *lb*, constraint in the *fmincon* command. Fig. 4 contains the simulation result where the number of impulses, *N*, was reduced to $N = 20$, which gives the input shaper length of 1.9. Other simulation parameters are the same as those used to produce Fig. 2(a)–(c) for the positive-impulse case. The output, *y*, and the shaped input, u_s, are given in Fig. 4(a) in the solid and dashed lines, respectively. The impulse amplitudes, A_i, are shown in Fig. 4(b), and the sensitivity curve is plotted in Fig. 4(c). One main advantage of using the negative-impulse input shaper is that it can significantly shorten the input shaper length. Due to the superposition principle, the length of the positive-impulse input shaper can never be shorter than the length of the ZV input shaper. However, this is not the case for the negative-impulse input shaper, and in fact the length can even be shorter than 1.9 and still producing good vibration suppression result. However, from Fig. 4, two disadvantages of the negative-impulse input shaper can be seen clearly. First, since the original input, *u*, is a unit-step input, from Fig. 4(a), the maximum value of the shaped input, u_s, exceeds that of the original input, which can lead to *overcurrenting*. Second, from Fig. 4(c), if the actual natural frequency is much larger than the model natural frequency $(\omega \gg \omega_n)$, the percentage vibration, *V*, can exceed 100%, which means the input shaper can even amplify the residual vibration, leading to instability.

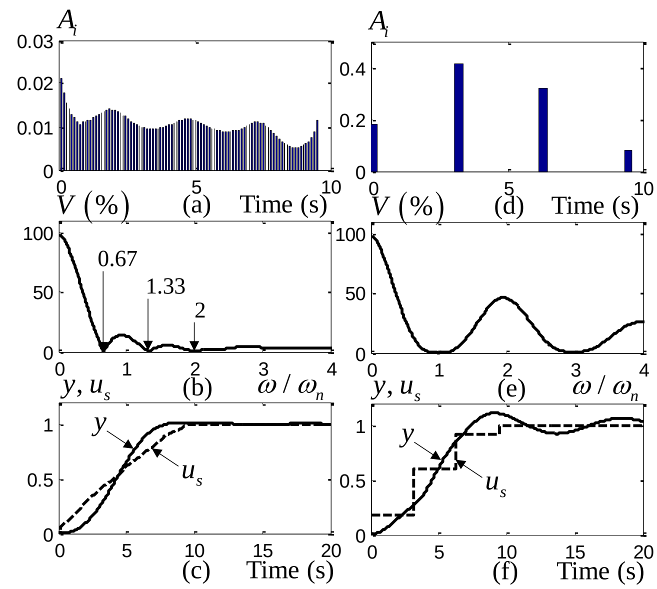

Fig. 3 Fixed-interval input shaper ((a)–(c)). ZVDD input shaper ((d)–(f)). (a), (d) Input shaper amplitudes. (b), (e) Sensitivity curve. (c), (f) Solid line is the output with the input shaper. Dashed line is the shaped input.

UNITY-MAGNITUDE INPUT SHAPER

A type of the negative-impulse input shaper is the *unity-magnitude* input shaper [12]. Its impulse amplitudes switch between 1 and -1, that is,

$$A_i = (-1)^{i+1} \text{ or } A_i = (-1)^i,\ i = 1, 2, \ldots, N,$$

for positive or negative movements, respectively.

For undamped second-order system, Ref. [12] proposed that the minimum number of impulses is three with time locations given by

$$t_1 = 0,\ t_2 = \pi / (3\omega_n),\ t_3 = 2\pi / (3\omega_n). \tag{18}$$

For underdamped second-order system, an optimization problem is required to solve for the time locations.

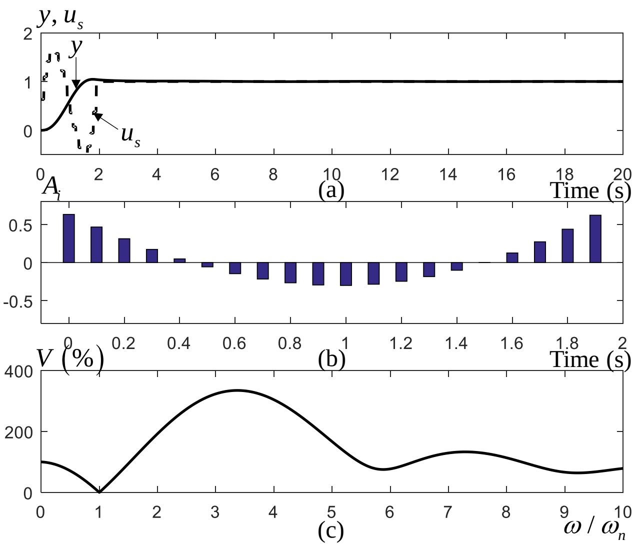

Fig. 4 (a) Solid line is the output with the input shaper. Dashed line is the shaped input. (b) Input shaper amplitudes. (c) Sensitivity curve.

For undamped fractional derivative system, Ref. [8] showed that the time locations of the three impulses are given by (18), with ω_n given by (9). For underdamped fractional derivative system, Ref. [8] proposed an approximated approach by assuming that the rise time of the unit-step response, t_r, is in the middle of t_2 and t_3 and that the unit-step response during t_2 and t_3 is a tangent line at the middle of the rise time, t_r. The resulting unity-magnitude is given by

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & t_r - \frac{1}{2\omega_n^2} y'(t_r) & t_r + \frac{1}{2\omega_n^2} y'(t_r) \end{bmatrix}, \tag{19}$$

where ω_n is given by (9), the rise time is

$$t_r = \frac{1}{\omega_n \sqrt{1-\zeta^2}} \left[\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta} \right) + \pi \right],$$

and $y'(t_r)$ is the derivative of the unit-step response at time $t = t_r$, given by

$$y'(t_r) = \omega_n e^{\left\{ -\frac{\zeta}{\sqrt{1-\zeta^2}} \left[\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta} \right) + \pi \right] \right\}}.$$

The proposed optimization problem to find a general unity-magnitude input shaper is as follows:

Find: $\Delta t_i,\ i = 1, 2, ..., N$

To minimize: $\sum_{i=1}^{N} \Delta t_i$

Subject to constraints:

$$\Delta t_1 = 0,$$

$$\Delta t_i \geq 0,\ i = 1, 2, ..., N,$$

$$V(\omega_j) < V_{\max}(j),\ j = 1, 2, ..., m, \tag{20}$$

where N is an odd number, $\Delta t_i = t_i - t_{i-1}$, m is the number of frequency points, and $V_{\max}(j)$ is the upper bound of the percentage vibration. The first constraint is for t_1 to start at zero. The second constraint ensures that the time locations will progress forward. The last constraint constrains the percentage vibration to a small value.

The command *fmincon* can be used to solve the optimization problem above by letting the optimization variable be $x = dt_i,\ i = 1, 2, ..., N$; the objective function be $f(x) = \sum_{i=1}^{N} dt_i$; the nonlinear inequality constraint be $c(x) = V - V_{\max}$; the linear equality constraint be $A_{eq} = [1, 0, ..., 0]$ and $b_{eq} = 0$; the lower bound, *lb*, be a vector of zeros; the initial guess, x_0, be a vector of ones; and the upper bound of the percentage vibration, $V_{\max}$, be zero.

Simulation was performed with the same fractional derivative plant as in previous sections, that is, the plant (6) with $n = 1.9$ and $\tau = 1$. The optimization problem above gave a three-impulse unity-magnitude input shaper whose amplitudes and time locations are given by

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1.2023 & 2.1046 \end{bmatrix}. \tag{21}$$

Fig. 5(a) shows the output in the solid line and the shaped input in the dashed line. Fig. 5(b) contains the input shaper amplitudes. Fig. 5(c) presents the sensitivity curve.

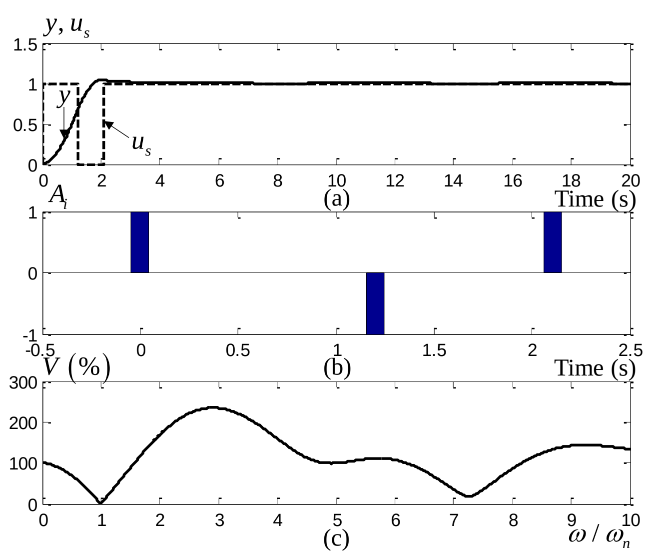

Fig. 5 (a) Solid line is the output with the input shaper. Dashed line is the shaped input. (b) Input shaper amplitudes. (c) Sensitivity curve.

By using (19), which was currently proposed by [8], the three-impulse unity-magnitude input shaper can be computed as

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1.2232 & 2.0951 \end{bmatrix}. \tag{22}$$

Comparing (22) to (21), it can be seen that the optimization problem can recover the work in [8].

Moreover, the proposed optimization problem can be used to solve for the unity-magnitude input shaper with more than three impulses. For example, when 7 impulses are used, that is, $N = 7$, with $\omega_j = [0.8\omega_n, \omega_n, 1.2\omega_n]$ in the percentage vibration constraint (20), the simulation result is shown in Fig. 6. More robustness can be obtained using the input shaper having more impulses.

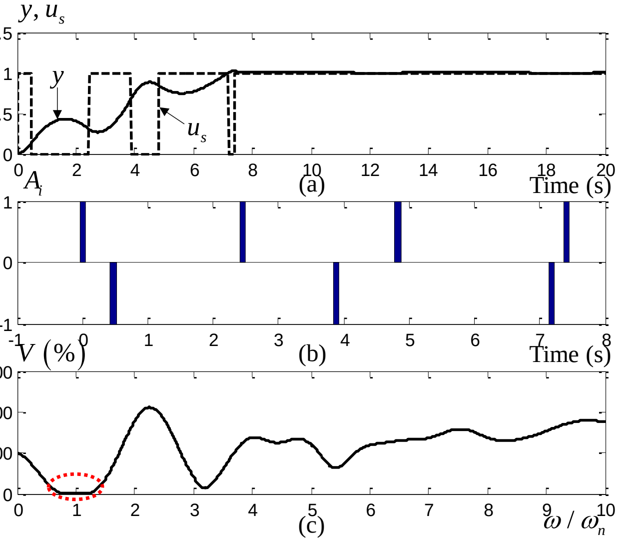

Fig. 6 (a) Solid line is the output with the input shaper. Dashed line is the shaped input. (b) Input shaper amplitudes. (c) Sensitivity curve.

SPECIFIED-INSENSITIVITY INPUT SHAPER

In designing the fixed-interval input shaper in Section IV, only the impulse amplitudes are the optimized variables because the impulse time locations are known and evenly spaced. In designing the unity-magnitude input shaper in Section V, only the impulse time locations are the optimized variables because the impulse amplitudes are known and switched between +1 and -1.

In this section, a type of input shaper, based on the *frequency sampling* technique, so-called *specified-insensitivity* input shaper [13], is designed for the fractional derivative system using nonlinear optimization.

In designing the specified-insensitivity input shaper, both the impulse amplitudes and time locations are to be found from the following optimization problem:

Find: $\Delta t_i, A_i, i = 1, 2, ..., N$

To minimize: $\sum_{i=1}^{N} \Delta t_i$

Subject to constraints:

$$\Delta t_1 = 0,$$

$$\Delta t_i \geq 0, i = 1, 2, ..., N,$$

$$V(\omega_j) < V_{\max}(j),\ j = 1, 2, ..., m \tag{23}$$

$$\sum_{i=1}^{N} A_i = 1,$$

$$A_i \geq 0, i = 1, 2, ..., N,$$

where N is an arbitrary number of impulses. Other variables are the same as those of the two previous input shapers.

The command *fmincon* can be used to solve the optimization problem above by letting the optimization variable be $x = [dt_i; A_i]$, $i = 1, 2, ..., N$; the objective function be $f(x) = \sum_{i=1}^{N} dt_i$; the nonlinear inequality constraint be $c(x) = V - V_{\max}$; the linear equality constraint be

$$A_{eq} = [1, \underbrace{0, ..., 0}_{2N-1}; \underbrace{0, 0, ..., 0}_{N}, \underbrace{1, 1, ..., 1}_{N}],\ \mathrm{b}_{eq} = [0; 1];$$

the lower bound, *lb*, and the initial guess, x_0, be two vectors of zeros; and the upper bound of the percentage vibration, $V_{\max}$, be zero.

Simulation was performed with the same fractional derivative plant as in previous sections, that is, the plant (6) with $n = 1.9$ and $\tau = 1$. The number of impulses, *N*, was set equal to 20. The sampling frequencies in the percentage vibration constraint (23) was set equal to

$$\omega_j = [0.8\omega_n, \omega_n, 1.2\omega_n, 2.5\omega_n, 2.75\omega_n, 3\omega_n], \tag{24}$$

which covers two ranges of frequencies. Fig. 7 contains the simulation result. The shaped input, u_s, and the output, *y*, are shown in Fig. 7(a), in the dashed line and solid line, respectively. The impulse amplitudes are shown in Fig. 7(b), and the sensitivity curve is contained in Fig. 7(c). From the sensitivity curve, the percentage vibration is attenuated well during the two ranges of the sampling frequencies (24). The minimum input shaper length was found to be $t_N = 10.427$ s.

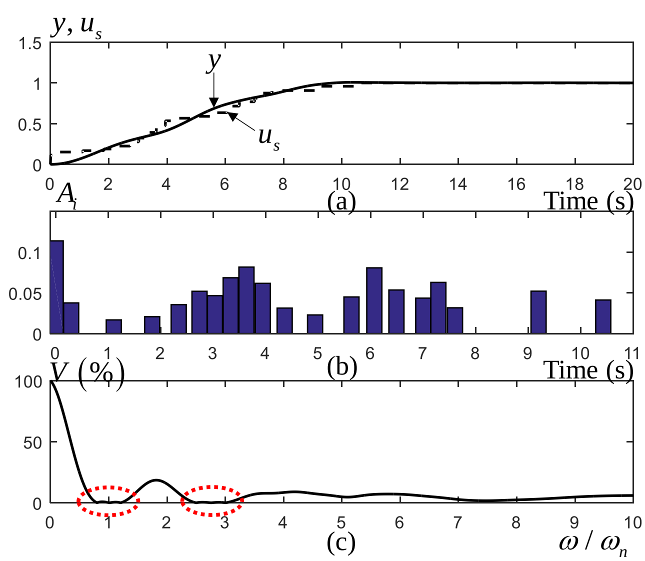

Fig. 7 (a) Solid line is the output with the input shaper. Dashed line is the shaped input. (b) Input shaper amplitudes. (c) Sensitivity curve.

CONCLUSIONS

Using nonlinear optimization, more complicated, better-performance input shapers are obtained for fractional derivative oscillatory systems. Fixed-interval input shaper can be designed to be robust to a wide range of parameter uncertainty and to attenuate high-frequency uncertainty. Its impulse can be made negative to shorten the input shaper length. For unity-magnitude input shaper, by using nonlinear optimization, more impulses can be added to the impulse sequence for more robustness. In specified-insensitivity input shaper, the input shaper length can be minimized whereas multiple modes of vibration can be suppressed.

Nonlinear optimization, presented in this paper, provides flexibility to extend existing input shapers to have additional features including to minimize the input shaper length of the fixed-interval input shaper, to eliminate overcurrenting and to limit high-mode excitation of the negative-impulse input shaper, and to be applicable to multivariable flexible system.

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