1 Introduction

Flexibility in the joint of a robot manipulator is intrinsic as well as by design. The joint flexibility arises from driving components such as actuators, gear teeth, or transmission belts. For safety, the joint flexibility is incorporated into the manipulator intentionally to absorb impact force to reduce damage. The experiment in Ref. [1] showed that the joint flexibility should be considered in both modeling and control design for the robot manipulator to obtain high performance and to avoid instability due to resonance.

To avoid resonance, input shaping is a technique that suppresses the residual vibration from moving flexible systems rapidly. The input shaping filter outputs a shaped signal that does not excite the flexible modes, resulting in zero residual vibration. The idea was originally proposed under the name Posicast control [2]. More robustness was added to the Posicast control by Singer and Seering [3], and the technique was patented under the name input shaping [4]. The technique was applied to a one-link flexible-joint robot manipulator. Singer and Seering [3] proposed the zero vibration and derivative (ZVD) input shaper, which is a simple zero vibration input shaper. It was shown that the proposed control system could significantly reduce the residual vibration, suppression in marine crane [4], liquid sloshing [5], flexible spacecraft [6], cam follower [7], hard disk drive [8], cherry picker [9], atomic force microscope [10], micromilling machine [11], quadrotor [12], fuel transport system of nuclear power plant [13], and automotive wiper [14].

The input shaping technique has also been applied to flexible-joint robot manipulators. Chatlatanagulchai and Saeheng [15] applied the zero vibration and derivative (ZVD) input shaper, proposed by Singer and Seering [3], to a one-link flexible-joint robot manipulator. The shaper was placed outside of the loop. A proportional-integral controller was used as feedback controller. Yun et al. [16] applied input shaping to a safe robot arm, having passive compliant joints. The robot arm has three links and three compliant joints. The natural period of the third link is constant whereas that of the second link varies, depending on the configuration of the third link. A curve fit between the third link's angle and the second link's natural period was applied to the second link. Similar procedure could also be applied to the first link. For a one-link flexible-joint robot manipulator, Ahmad et al. [17] proposed a composite control system, consisting of a proportional-derivative-type fuzzy logic controller and the zero-derivative-and-double-derivative input shaper. It was shown that the proposed composite control system could significantly reduce the residual vibration,
leading to better tracking. Pereira et al. [18] applied the algebraic identification technique, proposed by Fliess and Sira-Ramirez [19], to identify the first-mode natural frequency of a single-link flexible manipulator online. The natural frequency was fed to the ZVD input shaper to update it with new information. The algebraic identification technique has two advantages in that it requires short time to obtain the system parameters and no initial conditions are needed. For a two-link flexible-joint robot, Yavuz et al. [20] presented a twice-shaped method. A trajectory based on a cycloidal and versine functions was first created; then, the trajectory was reshaped again with an input shaper with two impulses. The benefit in doing so is vibration reduction in uncertain multimode systems.

Because the performance of the original Posicast control degrades when there is uncertainty in the mode parameters. Several researchers have proposed robust input shapers to increase its robustness. The robust input shaper, proposed by Singer and Seering [3], was so-called ZVD\textsuperscript{F} shaper. During its design, higher-order derivatives of the zero residual vibration expression, taken with respect to the mode parameters, were set equal to zero and used as additional constraints. If a small amount of residual vibration, rather than zero, is allowed when the system model is exactly known, the robustness of the input shaper can be substantially increased. The input shaper that uses this concept is a so-called extra-insensitive (EI) shaper, proposed by Singhose et al. [21]. Multiplication of a series of ZV input shapers in the Laplace domain was proposed in Ref. [22]. The impulse times of each ZV input shaper in the series were slightly perturbed, resulting in a so-called perturbation-based extra-insensitive input shapers (PEI-ISS).

Other robust input shapers use different concepts, including enforcing constraints on the vibration amplitude over a user-specified range of natural frequencies [23], solving an optimization problem to minimize the maximum magnitude of the residual states over a range of uncertain parameter values [24], considering the probability distribution of the natural frequency about its modeled value during the shaper design [25,26], and formulating the design problem as an optimization problem having linear matrix inequality constraints [27].

A limited number of researchers have used input shaping with model matching. Chattatanagulchai and Kaveesontornsanooh [28] employed the iterative learning controller to match the closed-loop system to a reference model. The ZV input shaper was placed outside of the closed-loop system and was designed from the reference model. However, this system is only suitable for robot tracking repeated path. Pai [29] and Hu et al. [30] used sliding-mode controllers in model matching. However, the plant and the reference model were assumed to be in linear, modal form. Applicable only to a simple transfer function plant, Yuan and Chang [31] proposed using two gains in a model matching with input shaping. Yu and Chang [32] applied a cascaded feedback control system. The outer loop controller is for model matching, whereas the inner loop is for stabilization. Also limited to linear, modal form plant, Fujioka and Singhose [33] proposed a Lyapunov-based feedback control law to model matching with input shaping.

Disadvantages of the input shaping techniques that have been used with the flexible-joint robots [15–20], the previously proposed robust input shapers [3,21–27], as well as the previously proposed input shaping with model matching [28–33] are that more robustness must come at the price of having longer move time, the plant must be linear- and time-invariant, and vibration induced by disturbance and noise cannot be suppressed.

This paper considers a system-based method of model matching. Closeness between the resulting closed-loop system and the reference model was formulated as a frequency-domain specification. This specification, together with other specifications including disturbance and noise rejection and stability, was converted to bounds on the Nichols chart. A suitable feedback controller and feed-forward filter were designed to satisfy these bounds. The ZV shaper, which has the shortest duration, was placed outside of the closed-loop system. It was then designed from the vibratory mode parameters of the reference model, which are known exactly.

The proposed system offers several advantages as follows:

- The system can tolerate large amount of uncertainty in the vibratory mode parameters. With 27 plant model variations, obtained from varying the spring constant, the deadzone and backlash parameter, and the payload mass moment of inertia, the proposed system can match the closed-loop system with one fixed reference model well, according to a prespecified model matching specification. As a result, the robot link can track its desired trajectory well with no residual vibration for all the 27 plant model variations. The proposed system is literally insensitive to uncertainty in the natural frequency or damping ratio.
- The shaped reference signal for the flexible-joint robot has the shortest possible duration, and the duration does not increase with the amount of robustness. The quantitative feedback controller handles the uncertainty; therefore, the simplest ZV input shaper with the shortest length can be used. When the amount of uncertainty increases, the ZV input shaper still applies, so the duration does not increase.
- The applicable plant can be nonlinear or time-varying systems. When payload changes with time, more residual vibration can be seen with traditional robust input shapers. However, the residual vibration remains zero with the proposed system as long as the variation is not outside the design limits. Nonlinear spring, deadzone, and backlash were found to degrade the input shaping performance. However, the degradation was not noticed using the proposed system.
- The system can also suppress vibration induced by disturbance and noise. The proposed system can suppress vibration induced by plant-input disturbance, plant-output disturbance, and noise for all the 27 plant model variations. Nevertheless, the amount of noise attenuation must be traded for stability margin.

The paper is organized as follows: Section 2.1 presents details on the flexible-joint robot manipulator of interest. The details include experimental setup, nonlinear governing equations,
linearized mathematical model, and preliminary simulation result using only proportional control. Section 2.2 contains brief discussions on input shaping technique and the three types of robust input shapers that were compared to the proposed system. They are the ZVD$^k$ input shaper, the EI input shaper, and the PEI-IS. Section 2.3 discusses the proposed quantitative feedback input shaping (QF-IS) system. The details include 27 plant model variations, frequency-domain specifications, and open-loop shaping. Simulation and experimental results are given in Sec. 3. Conclusions are drawn in Sec. 4.

2 Materials and Methods

2.1 Flexible-Joint Robot Manipulator

2.1.1 Experimental Setup. In this paper, a flexible-joint robot, as shown in Fig. 1, was used in the experiment. Its mathematical model was used in the simulation. A time-varying payload and an accelerometer (not shown in the figure) were attached to the link tip. Two optical encoders were used to measure the motor angle $\theta_2$ and the link angle relative to the motor $\theta_1 - \theta_2$. Two soft springs were attached to the link and the motor hub to provide flexibility.

Figure 2 shows a diagram of the experimental setup. A host computer, with pertaining software, communicates with the user and a target computer. The target computer contains a National Instruments’ data-acquisition card, modeled PCI-6221, whose functions are to acquire sensor signals and to send out actuator command from the control algorithm. The host and target computers are connected to each other via a local area network cable. Control signal is sent as voltage to a dimension engineering’s motor amplifier board, modeled Sabertooth 2 x 25, to be amplified to a level that can drive the direct current (DC) motor. A Pololu’s accelerometer, modeled MMA7341LC, is mounted at the link tip to measure linear acceleration $\ddot{x}$. A DC power supply supplies 24 V current to the motor amplifier board.

2.1.2 Nonlinear Governing Equations. A top-view diagram of the flexible-joint robot is given in Fig. 3. $\theta_1$ and $\theta_2$ are the absolute angular position of the link and motor hub.
From free-body diagram of the link, \( M_k \) is the moment of the nonlinear spring:

\[
(J + J_p)\dddot{\theta}_1 = -c_1(\dot{\theta}_1 - \ddot{\theta}_2) + M_k
\]

\[
M_k = -k_s(L_1 - L)(\cos \alpha)r_1 \cos(\theta_1 - \theta_2)
\]

\[
+ k_s(L_1 - L)(\sin \alpha)r_1 \sin(\theta_1 - \theta_2)
\]

\[
+ k_s(L_2 - L)(\cos \beta)r_1 \cos(\theta_1 - \theta_2)
\]

\[
+ k_s(L_2 - L)(\sin \beta)r_1 \sin(\theta_1 - \theta_2)
\]

\[
L_1 = \sqrt{\frac{r_1 \cos(\theta_1 - \theta_2) - r_3}{r_1 \sin(\theta_1 - \theta_2) + r_2}}^2
\]

\[
L_2 = \sqrt{\frac{r_1 \cos(\theta_1 - \theta_2) - r_3}{-r_1 \sin(\theta_1 - \theta_2) + r_2}}^2
\]

\[
\alpha = \tan^{-1}\left(\frac{r_1 \cos(\theta_1 - \theta_2) - r_3}{r_1 \sin(\theta_1 - \theta_2) + r_2}\right)
\]

\[
\beta = \tan^{-1}\left(\frac{r_1 \cos(\theta_1 - \theta_2) - r_3}{-r_1 \sin(\theta_1 - \theta_2) + r_2}\right)
\]

From free-body diagram of the motor hub:

\[
J_0\dddot{\theta}_2 = c_1(\dot{\theta}_1 - \ddot{\theta}_2) - M_k - c_2\ddot{\theta}_2 + T_3
\]

Backlash model:

\[
\dot{T}_3 = B(T_3, T_2, \dot{T}_2) = \begin{cases} 
 m\dot{T}_2, & \text{if } \dot{T}_2 > 0 \text{ and } T_3 = m(T_2 - b^-) \text{ or } \dot{T}_3 > 0 \\
 0, & \text{otherwise}
\end{cases}
\]

Deadzone model:

\[
T_2 = D(T_1) = \begin{cases} 
 m^-(T_1 - d^-), & T_1 \leq d^- \\
 0, & d^- < T_1 < d^+ \\
 m^+(T_1 - d^+), & T_1 \geq d^+
\end{cases}
\]

DC motor electrical model:

\[
v = Ri + L_m \frac{di}{dt} + k_i \dot{\theta}_2
\]

\[
T_1 = k_i i
\]

Power amplifier:

\[
v = k_u u
\]
\( x, \beta, L_1, L_2, r_1, r_2, \) and \( r_3 \) are the related dimensions. From direct measurements and system identifications, parameter values of this flexible-joint robot are given in Table 1.

Using the Lagrange–Euler method and physical laws, the nonlinear governing equations of the flexible-joint robot are given in Table 2.

\( u(\pm 2.5 \text{ V}) \) is the control command voltage from the data-acquisition card to the motor driver. \( v(\pm 24 \text{ V}) \) is the amplified input voltage from the motor driver to the motor. \( i \) and \( T_1 \) are the current in the motor coil and the torque produced by the motor coil. In the simulation, deadzone and backlash models, given in Ref. [34], were added to represent hard nonlinearities. \( T_2 \) is the torque output from deadzone, and \( T_3 \) is the torque output from backlash to the motor hub. The nonlinear governing equations in Table 2 represented the actual robot in the simulation.

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2.1.3 Linearized Mathematical Model. Neglecting the deadzone and backlash models and assuming zero motor coil inductance \( (L_m = 0) \), the nonlinear governing equations of the flexible-joint robot can be linearized about \( \theta_1 - \theta_2 = 0 \) as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & k_{slin} & 1 & 0 & 0 \\
k_{slin} & J_1 + J_2 & J_1 + J_2 & J_1 + J_2 & 0 \\
0 & 0 & 0 & J_2 & 0 \\
k_{slin} & J_h & J_h & J_h & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
k_{slin} \\
k_{slin}
\end{bmatrix}
\begin{bmatrix}
u \\
u
\end{bmatrix}
\]

(1)

where \( x_1 = \theta_1, x_2 = \dot{\theta}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2 \), and

\[
k_{slin} = \left. \frac{\partial M_k}{\partial (\theta_1 - \theta_2)} \right|_{\theta_1 - \theta_2 = 0} = 8.043
\]

(2)

is the linearized spring constant. Substituting the parameter values in Table 1 into Eq. (1), the vibratory mode parameters, which are the natural frequency and damping ratio, can be calculated from the pole locations of the linear system as

\[
\omega_n = 15.9 \text{ rad/s}, \quad \zeta = 0.295
\]

(3)

The reference model, used in the proposed control system, was a second-order oscillator

\[
F_m(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

(4)

where the natural frequency \( \omega_n \) and the damping ratio \( \zeta \) were obtained from linearizing the nonlinear governing equations of the robot, given by Eq. (3).
The linearized mathematical model and the reference model were used in the design of the proposed control system.

2.1.4 Preliminary Result With Proportional Control. Without input shaping, the nonlinear governing equations in Table 2 were simulated using a proportional controller with a gain \( K_p = 0.03 \) and a baseline reference signal \( r_b \) as a square wave of amplitude 1 rad. Figure 4 contains the simulation result.

Without input shaping, a substantial amount of residual vibration can be seen from the link position \( \dot{\theta}_l \) as shown in Fig. 4(a) because the square reference signal excites the vibration mode of the flexible-joint robot. Persisting residual vibration of a small but constant amplitude is also an evidence of the presence of unattended backlash nonlinearity. The corresponding control input \( u \) is shown in Fig. 4(b), which is still well within the ±2.5 V limits, which means that the gain \( K_p \) can still be increased, causing more residual vibration.

In the linearized state-space model of the robot (1), a linearized spring constant \( k_{slin} \) is plotted as a function of \( \theta_1 - \theta_2 \) as shown in Fig. 4(c) with a range of 7.359 \( \leq k_{slin} \leq 12.43 \).

The deadzone with \( b_- = -0.5 \text{ N} \cdot \text{m} \) and \( b_+ = 0.5 \text{ N} \cdot \text{m} \) is evident in the plot between the torque produced by the motor coil \( T_1 \) and the torque output from deadzone \( T_2 \), as shown in Fig. 4(d). The backlash with \( d_- = -0.5 \text{ N} \cdot \text{m} \) and \( d_+ = 0.5 \text{ N} \cdot \text{m} \) is also shown in the plot between \( T_2 \) and the torque output from backlash \( T_3 \) in Fig. 4(e).

2.2 Input Shaping. Input shaping technique [35] computes amplitudes and time locations of an impulse sequence, so that when it is applied to a linear flexible system, its impulse responses cancel out, resulting in zero residual vibration.

2.2.1 ZVD\(^k\) Input Shaper. In general, a ZVD\(^k\) input shaper, where \( k = 0, 1, 2, \ldots \), has the total of \( k + 2 \) impulses in the sequence and is shown to have normalized impulse amplitudes and timings as

\[
A_i = \left( \frac{k + 1}{i - 1} \right)^{i-1}, \quad t_i = (i - 1) \frac{\pi}{\omega_0 \sqrt{1 - \xi^2}},
\]

\[i = 1, 2, \ldots, k + 2\]

where \( \omega_0 \) is the natural frequency of the applied linear system, \( \xi \) is its damping ratio, \( t_i \) is the time the \( i \)th impulse is applied, and \( A_i \) is the \( i \)th impulse's amplitude

\[
K = e^{-\xi \sqrt{\eta - \xi^2}}
\]

and

\[
\binom{n}{r} \equiv \frac{n!}{r!(n-r)!}
\]

is the combinations of \( n \) things taken \( r \) at a time.

2.2.2 EI Input Shaper. Singhose et al. [21] allowed a low level of vibration \( V_{lim} \) at the modeling frequency, creating a so-called EI shaper, which is more robust than the ZVD\(^k\) shaper, for the same shaper duration.

For damped systems, the impulse amplitudes and timings cannot be obtained in closed form. They must be solved numerically. However, a surface fit can be used to obtain their values in terms of \( V_{lim} \), \( \xi \), and \( T_d = 2\pi/(\omega_0 \sqrt{1 - \xi^2}) \). Singhose et al. [36] presented these relationships.

2.2.3 Impulse-Time Perturbation Input Shaper. Rew et al. [22] proposed a so-called PEI-ISs. A one-hump PEI-IS is given by

\[
F_{12}(s) = F_1(s)F_2(s) = [A_1 + A_2e^{-\tau_1(1-i\omega)}][A_1 + A_2e^{-\tau_1(i+1\omega)}]
\]

where \( 0 < e < 1 \) is a perturbation parameter, which can be found from a relationship

\[
e = \frac{0.9981 \sqrt{V_{lim}(1-\xi^2)}}{\pi A_2}
\]

A two-hump PEI-IS is given by

\[
F_{012}(s) = F_0(s)F_1(s)F_2(s) = [A_1 + A_2e^{-\tau_1eT_2}][A_1 + A_2e^{-\tau_1(1-i\omega)}][A_1 + A_2e^{-\tau_1(i+1\omega)}]
\]

A three-hump PEI-IS is given by

\[
F_{123a}(s) = F_1(s)F_2(s)F_3(s)F_4(s)
\]
where

\[ F_3(s) = A_1 + A_2 e^{-\tau_1(1-\delta) s} \quad F_4(s) = A_1 + A_2 e^{-\tau_2(1+\delta) s} \]

and \( \delta \) is an additional design parameter.

### 2.3 QF-IS

#### 2.3.1 Plant Templates

Quantitative feedback controller [37] is designed for all the possible plant cases. Each case represents one set of the plant parameter values. Because the spring is non-linear, the linearized spring constant was set to three values

\[ k_{s\text{lin}} = \{7.359, 8.043, 12.43\} \text{ N} \cdot \text{m} \quad (7) \]

which covers possible range of \( k_{s\text{lin}} \), as shown in Fig. 4(c). The deadzone and backlash alter the shape of the control input. Their effect can be accounted for by multiplying the term \( k_d / J_h R \) in Eq. (1) by a constant that was set to three possible values

\[ k_d = \{0.8, 1, 1.2\} \quad (8) \]

To simulate the time-varying property of the robot, the payload was allowed to change within \( \pm 50\% \) of its nominal value in Table 1. Therefore, the payload mass moment of inertia was allowed to have three possible values

\[ J_p = \{0.014, 0.028, 0.042\} \text{ kg m}^2 \quad (9) \]

From Eqs. (7) to (9), there are \( 3 \times 3 \times 3 = 27 \) possible plant cases as given by the linear state-space plant model \( P_{\text{lin}} \). Figure 6 shows plant templates of all the 27 plant cases for eight frequencies of interest \( \omega = \{0.05, 1, 2, 5, 10, 16, 20, 40\} \) rad/s, which cover low, high, and natural frequencies of the plant. The control system will ensure that the closed-loop system will meet various frequency-domain specifications for all the 27 plant cases and for all the eight frequencies. The nominal plant case \( P_{\text{nom}} \) is the fourteenth case, where all the three parameters have their middle values.

#### 2.3.2 Frequency-Domain Specification

A diagram of the proposed QF-IS system is given in Fig. 7, where \( r_b \) is the baseline reference signal, IS is the ZV input shaper, \( r_s \) is the shaped reference signal, \( F \) is the prefilter to be designed, \( F_m \) is the reference model, \( e \) is the tracking error, \( G \) is the feedback controller to be designed, \( d_i \) is the plant-input disturbance, \( u \) is the control input, \( P \) represents the 27 plant cases, \( d_o \) is the plant-output disturbance, \( y \) is the output, and \( n \) is the noise.

Fig. 8 Open-loop shaping: (top) original shape and (bottom) after shaping

Fig. 9 Diagram of the traditional robust input shaping

Fig. 10 Bode magnitude plots: (top) the proposed QF-IS system and (bottom) traditional robust input shaping system

\[ F_3(s) = A_1 + A_2 e^{-\tau_1(1-\delta) s}, \quad F_4(s) = A_1 + A_2 e^{-\tau_2(1+\delta) s} \]
Several frequency-domain specifications were imposed.

1. Stability margin specification is given by

\[ \frac{y}{n} = \frac{PG}{1 + PG} < \delta_n \]  \hspace{1cm} (10)

where \( \delta_n \) was set equal to 3 dB or \( 10^{1.20} = 1.413 \). It was shown in Ref. [38] that this specification is equivalent to the gain margin of

\[ GM = 20 \log \left[ \frac{\delta_n + 1}{\delta_n} \right] = 4.65 \text{ dB and the phase margin of} \]

\[ PM = 2 \sin^{-1} \left[ \frac{1}{\sqrt{2} \delta_n} \right] = 0.724 \text{ rad} = 41.46 \text{ deg}. \]

2. Disturbance rejection specifications are

\[ \frac{y}{d_I} = \frac{P}{1 + PG} < \delta_{dI} \]  \hspace{1cm} (11)

for the plant-input disturbance and

\[ \frac{y}{d_O} = \frac{1}{1 + PG} < \delta_{dO} \]  \hspace{1cm} (12)

3. Model matching specification is given by

\[ \left| \frac{y}{r_I} - F_m \right| = \left| \frac{PGF}{1 + PG} - F_m \right| < \delta_m \]  \hspace{1cm} (13)

where \( F_m \) is the reference model (4), and \( \delta_m \) was set equal to 20 dB or \( 10^{-10/20} = 0.316 \).

The specifications were converted into bounds on the Nichols chart. The open-loop plot was shaped to be in the allowable regions of the bounds. The details are given in the next section.

2.3.3 Open-Loop Shaping. For each frequency of interest \( \omega_i \), the specifications (10)–(13) were converted into allowable regions with bounds on the Nichols chart. The open-loop plot was shaped to be in the allowable regions of the worst-case bounds.

Figure 8 (top) shows the worst-case bounds from specifications (10)–(13) on the Nichols chart for the frequencies of interest, \( \omega = \{0.05, 1, 2, 5, 10, 16, 20, 40\} \) rad/s. Figure 8 (top) also contains the original open-loop shape \( L(s) = P(s)G(s) \), where \( G(s) = 1 \) and \( P(s) \) is the linear model of the robot as given in Eq. (1).
To satisfy all the bounds and hence the specifications (10)–(13), the open-loop shape must be shaped so that at each frequency, the open-loop shape lies above or outside the corresponding bounds. To do so, a real zero was appended to $G(s)$ to increase the open-loop gain and phase, that is, to move the open-loop shape to the right. In addition, a complex pole was appended to $G(s)$ to roll off the open-loop shape at high frequencies so that the high-frequency noise can be attenuated. The complex pole was selected because it decreases the open-loop phase but does not significantly affect the open-loop gain. Finally, the gain of $G(s)$ was adjusted so that all the bounds were satisfied. The final feedback controller $G(s)$ is then given by

$$G(s) = 3.78 \left( \frac{s}{12.58} + 1 \right) \left( \frac{1}{s^2 + 2(0.354)s + 897.57^2} + 1 \right)$$

The final open-loop shape is shown in Fig. 8 (bottom), where all the bounds are satisfied. The readers who are not familiar with quantitative feedback control are suggested to consult Ref. [39] and references therein.

The prefilter $F(s)$ was chosen as the reference model (4), that is,

$$F(s) = F_m(s) = \frac{15.9^2}{s^2 + 2(0.295)(15.9)s + 15.9^2}$$

so that the steady-state gain is one and the model matching specification (13) becomes the plant-output disturbance rejection specification (12) for which standard loop shaping algorithm is available.

### 3 Results and Discussion

In this section, the proposed QF-IS system (Fig. 7) is compared with traditional robust input shapers in Sec. 2. The traditional robust input shaping system is given in Fig. 9.

Both simulation and experimental results are included in this section. The experiment was performed on the one-link flexible-joint robot manipulator as shown in Fig. 1. In the simulation, either the nonlinear robot model in Table 2 or the 27 linear plant cases given by Eq.(1) was used to represent the actual robot.

For the proposed QF-IS system, the controller $G(s)$ and prefilter $F(s)$ are given by Eqs. (14) and (15). The ZV input shaper is given in Sec. 2.2.1, designed using the vibratory mode parameters (3).

For the traditional robust input shaping system, the proportional controller $G(s)$ is the same gain $K_p = 0.03$ as that of Sec. 2.1.4. The robust input shapers are the ZVD input shaper in Sec. 2.2.1, the EI input shaper in Sec. 2.2.2, and the impulse-time perturbation input shaper in Sec. 2.2.3.

#### 3.1 Tolerate Large Amount of Uncertainty

For the proposed QF-IS system (Fig. 7), the ZV input shaper is designed from the vibratory mode parameters of the reference model $F_m$. The model matching specification (13) ensures that the closed-loop system from $r$ to $y$ is close to the reference model $F_m$ for all the 27 plant cases. Figure 10 (top) contains this result where the solid lines are the magnitude of the mapping from $r$ to $y$ for all the 27 plant variations, and the dotted line is the magnitude of the reference model $F_m$. It can be seen that even though the plant $P$ is uncertain, the quantitative feedback controller still keeps the closed-loop magnitude close to the reference model $F_m$. Therefore, the input shaper need not be robust, and the simplest and shortest ZV input shaper can be used.

For the traditional robust input shapers (Fig. 9), the robust input shapers are designed from the vibratory mode parameters of the closed-loop mapping, which is $PG/(1 + PG) = P_{nom}K_p/(1 + P_{nom}K_p)$, where $P_{nom}$ is the nominal plant and $K_p$ is the proportional gain. Figure 10 (bottom) shows the bode magnitude plots of $P_{nom}K_p/(1 + P_{nom}K_p)$ and the mapping from $r$ to $y$ for all the 27 plant variations. It can be seen that the mapping from $r$ to $y$ is not necessarily close to $P_{nom}K_p/(1 + P_{nom}K_p)$. Therefore, when the plant is uncertain, the input shapers in this case must be robust, resulting in longer move time.

In the time-domain, Fig. 11 contains the link angular position $\theta_1$, where the baseline reference $r_0$ is a square wave of amplitude 1 rad. Figure 11 (top) shows the result of the traditional robust input shaping system (Fig. 9) using the ZVD input shaper. The ZVD input shaper was designed using the mode parameters of the closed-loop system having the nominal plant $P_{nom}$. Therefore, when the plant varies, as in the 27 plant cases, its performance in suppressing the residual vibration deteriorates. Figure 11 (bottom) illustrates the tracking result of the proposed QF-IS system (Fig. 7). For all the 27 plant variations, the ZV input shaper is still able to suppress the residual vibration perfectly. This is because the quantitative feedback control system was designed to match the closed-loop system to a fixed reference model, regardless of the plant uncertainty.

![Fig. 14 Comparison of input shaper lengths](image1)

![Fig. 15 Link position tracking experimental result for nonlin-ear plant with time-varying payload: (top) traditional robust input shaping system with ZV input shaper and (bottom) proposed QF-IS system](image2)
Fig. 16 Simulation result of link position tracking: (a) traditional ZV input shaper with linear plant, (b) traditional ZV input shaper with nonlinear plant, (c) proposed QF-IS with linear plant, and (d) proposed QF-IS with nonlinear plant.

Fig. 17 Simulation result of link position under step disturbance: (a) traditional ZVD input shaper under step plant-output disturbance, (b) proposed QF-IS under step plant-output disturbance, (c) traditional ZVD input shaper under step plant-input disturbance, and (d) proposed QF-IS under step plant-input disturbance.
To evaluate the robustness of the input shaping systems, a formula of the percentage vibration obtained from step responses is given by Vyhlidal et al. [40]

\[
V(\omega_i, c) = \frac{\max(h_G(t)) - h_G(\infty)}{\max(h_G(t)) - h_G(\infty)} \times 100
\]

where \(h_G(t)\) is the step response of the system without the input shaper, and \(h_G(t)\) is the step response with the input shaper.

Figure 12 shows sensitivity curves of the traditional robust input shaping system (Fig. 9) with the ZV and ZVD (5), one-hump EI (Sec. 2.2.2), and one-hump PEI-IS (6) shapers, compared with that of the proposed QF-IS system (Fig. 7) with the ZV shaper. It can be seen from Fig. 12 (left) that, among the traditional input shapers, the PEI-IS is the most robust with respect to uncertainty in the model natural frequency. From Fig. 12 (right), the ZVD shaper is the most robust with respect to uncertainty in damping ratio. However, the proposed QF-IS system is not sensitive to either the uncertainty in the model natural frequency or damping ratio. This is because the QF-IS system is designed on a fixed reference model. The uncertainty is handled by the quantitative feedback control. Figure 13 contains the experimental verification of the simulation result, presented previously in Fig. 12. The experimental percentage vibration is not exactly zero due to some experimental factors that were not accounted for. However, the experimental percentage vibration remains small and flat across the uncertain range of interest.

### 3.2 Shorten Shaped Reference Duration.

The shaper length of the proposed QF-IS system equals that of the ZV input shaper, which has the shortest length. The length also does not increase with insensitivity as can be seen in Fig. 14. The insensitivity is an important measure of robustness in an input shaper. The 10% insensitivity is the width of the normalized frequency in the sensitivity curve such that the percentage vibration remains below 10%. Therefore, the larger the insensitivity, the more robust the input shaper. In the QF-IS system, because the quantitative feedback controller handles the uncertainty, the input shaper length remains unchanged and equals that of the ZV shaper.

Figure 14 also shows that, among the traditional robust input shapers with the same input shaper length, the EI and the ZVD\(^S\) shapers are the most robust.

### 3.3 Applying to Nonlinear or Time-Varying Systems.

Figure 15 shows the experimental result of link position tracking under time-varying payload. The payload was varied linearly from its nominal value of \(J_p = 0.0281 \text{ kg} \cdot \text{m}^2\) to \(J_p = 0.0843 \text{ kg} \cdot \text{m}^2\) during an interval of 40 s, which is an increase of 300%. Figure 15 (top) contains the result of using the traditional robust input shaping system (Fig. 9) with ZV input shaper. It can be seen that the vibration suppression performance degrades when the payload is time-varying, and its value moves away from its nominal value. Figure 15 (bottom) presents the result of using the proposed QF-IS system (Fig. 7). Even though the quantitative feedback control was designed to withstand 50% of payload variation from the nominal value, the vibration suppression performance is still acceptable during 300% payload variation with substantially less vibration than that of the traditional robust input shaping case.

Figure 16 contains comparison simulation result of link position tracking when nonlinearity is present. Figure 16(a) presents the result of using traditional ZV input shaper (Fig. 9) with linear plant (1). The residual vibration is totally suppressed because the ZV input shaper was also designed from the same linear plant (1). Figure 16(b) shows the result of using traditional input shaper (Fig. 9) with full nonlinear plant in Table 2. It can be seen that the nonlinear spring, deadzone, and backlash cause residual vibration. Figures 16(c) and 16(d) provide the result of using the proposed QF-IS system (Fig. 7) with linear plant (1) and nonlinear plant in Table 2, respectively. It can be seen that the nonlinearity has no effect on the performance of the input shaping system because the quantitative feedback controller was designed to handle the nonlinearity. The reason for this can be seen in the frequency-domain. Figure 18 shows Bode magnitude plots of the mapping from \(d_t\) to \(y\) (Fig. 18 (top)) and from \(d_o\) to \(y\) (Fig. 18 (bottom)) for all the 27 plant cases. Their magnitudes are well within the specifications (11) and (12) as was designed for. Note that the limits \(\delta_{ai} = \delta_{a0} = \)

\[
\delta_{ai} = \delta_{a0} = \frac{1}{1 + PG} \quad (\text{dB})
\]

![Fig. 18 Disturbance rejection: (top) plant-input disturbance rejection and (bottom) plant-output disturbance rejection](http://dynamicsystems.asmedigitalcollection.asme.org/pdfaccess.ashx?url=/data/journals/jdsmaa/935118/ on 07/30/2017 Terms of Use: http://www.asme.org/...
\[ -10 \text{dB} = 0.316 \text{ are still conservative because only around 70}\% \text{ of the disturbance was aimed to be rejected. This is the case for the plant-input disturbance rejection (Fig. 17(d)), where around 0.3 rad. still remain in the output } \theta_1 \text{ as a result of the unit-step plant-input disturbance. However, for the plant-output disturbance rejection (Fig. 17(b)), the system can substantially suppress the plant-output disturbance. This is because, when } F = P_{mv}, \text{ the plant-output disturbance rejection specification (12) has the same form as the model matching specification (13), which has stricter limit of } \delta_m = -20 \text{dB = 0.1.}

To suppress vibration induced by the noise, the specification (10) can be imposed but with \( \delta_m < 1 \). However, this contradicts the stability margin specification where \( \delta_m \) is required to be greater than one. Therefore, a tradeoff must be quantitatively evaluated between these two specifications. In our work, more emphasis was placed on the stability margin.

### 4 Conclusions

The proposed technique matches the closed-loop system to a fixed reference model. The design was performed on a set of uncertain plant models that covers all foreseeable uncertain plant, time-varying plant, as well as nonlinear plant. Tradeoffs, among stability margin, disturbance rejection, model matching, tracking, and noise rejection, are quantifiable with this technique. As a result, the input shaper, placed outside of the closed-loop system, can be kept as simple as possible, resulting in the shortest robot move time.

With this technique, the flexible-joint robot manipulator was found to move faster, to tolerate larger amount of uncertainty, and to have less residual vibration induced by reference, disturbance, and noise. The technique is also applicable to nonlinear or time-varying systems as long as the system varies within the prescribed limits. A future work includes extending the technique to multimode systems such as flexible-link robot manipulator or double pendulum crane model and to multi-input multi-output systems. Multi-mode systems currently require multiple input shapers, usually one shaper per one mode, resulting in substantially slow moving time of the robot. The control input usage by the proposed technique is expected to be less than that of the traditional technique due to less vibration occurred in the output. This will also be investigated in our future work.

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### Nomenclature

- \( A_i \): impulse amplitudes
- \( d_i \): plant-input disturbance
- \( d_o \): plant-output disturbance
- \( e \): tracking error
- \( F \): prefilter
- \( F_{in} \): reference model
- \( G \): controller
- \( K_p \): proportional gain
- \( k_{slim} \): linearized spring constant
- \( n \): noise
- \( P \): plant
- \( P_{lin} \): linear state-space plant model
- \( P_{nom} \): nominal plant
- \( r_b \): original reference input
- \( r_s \): shaped reference input
- \( t_i \): time locations of the impulses
- \( T_d \): damped period
- \( T_i \): torque produced by the motor coil
- \( T_o \): torque output from deadzone
- \( T_s \): torque output from backlash
- \( u \): control input
- \( v \): motor input voltage
- \( V \): percentage vibration
- \( V_{lim} \): percentage vibration limit
- \( x_i \): ith state variable
- \( y \): plant output
- \( \delta \): upper limit of frequency-domain specifications
- \( \varepsilon \): perturbed parameter
- \( \theta_1 \): absolute link angular position
- \( \theta_3 \): absolute motor angular position
- \( \zeta \): damping ratio
- \( \omega_n \): natural frequency

### References


[38] Chatlatanagulchai, W., 2011, System Modeling and Control, Misterkopy, Bangkok, Thailand, Chap. 10.
