TSF002

Fin Temperature Estimate and Model Predictive Control with Constraints for Thermal Plate System

Poom Jatunitanon[1,2*], Sajcha Tawaytibhong[1], Sarawoot Watechagit[2]
and Withit Chartlatanagulchai[1]

[1] Control of Robot and Vibration (CRVLAB), Department of Mechanical Engineering, Faculty of Engineering,
Kasetsart University, 50 Ngam Wong Wan Rd, Lat Yao, Chatuchak, Bangkok, 10900
[2] Center for Energy Research and Testing Laboratory (CERT Lab), Department of Mechanical Engineering, Mahidol University,
25/25, Phutthamonthonsai 4 Rd, Salaya, Phutthamonthon, Nakronphatom, 73170
*Corresponding Author: poom_legend@hotmail.com, Telephone Number0-2942-8567 ext506

Abstract

Thermoelectric module is a heat pump device. When DC voltage is applied to the device, the substrate surface becomes cold for absorbing heat energy and another substrate surface becomes hot for releasing the heat energy. The heat sink is attached to the hot side, and the accumulated heat in the heat sink is removed by an AC fan. The main disturbance source of cold plate surface temperature is this accumulated heat; therefore, reduction of the accumulated heat from the heat sink can improve the tracking performance on cold side temperature of thermoelectric plate. The thermoelectric system has two inputs and two outputs. The first input is a voltage applied to power drive to operate thermoelectric and the second input is a PWM signal applied to solid state relay for adjusting the speed of AC fan. This paper proposes model predictive control for controlling cold plate temperature and setting a constraint for bounded estimated heat sink temperature. The heat sink temperature is estimated by using Kalman filter. Simulation and experimental results show that the proposed method handles the accumulated heat in the heat sink better than a model predictive controller without constraint, resulting in significantly more accurate tracking of the cold plate temperature.

***Keywords**:* Thermoelectric module, Model predictive control, Kalman filter, Heat sink.

1. Introduction

The thermoelectric device is a kind of heat pump device and has been applied to many heat transfer applications such as, laser diode coolers, heat exchanger, liquid cooling system and portable refrigerators.

When DC voltage is applied to thermoelectric device, the substrate surface becomes cold for absorbing heat energy and another substrate surface becomes hot for releasing the heat energy.

However, the drawback of thermoelectric is an accumulated heat from the hot side of thermoelectric plate.

The main disturbance source for tracking a cold side temperature caused by The heat stored from hot side temperature in a heat sink. Therefore, the reduction of heat stored in a heat sink can improve the tracking performance on cold side temperature of thermoelectric plate.

The Ac fan is used to remove an accumulated heat from a heat sink by adjusting the speed of a fan.

The test system of thermoelectric with fan can be shown in Fig 1.

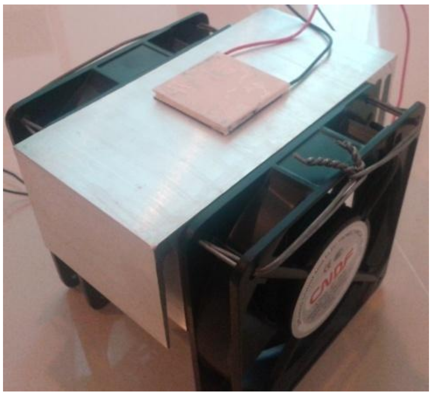

Fig. 1 Thermoelectric with fan system

Several techniques have been proposed in the literature for controlling a cold side temperature of a thermoelectric plate, for example, robust control, fuzzy control and model predictive control without constraints in difference applications.

In [2-4] have proposed the fuzzy control of the thermal plate system for dealing with nonlinear behavior of the thermal plate such as, thermal mass, ambient temperature and cooling load of thermal plate device.

Controller based on robust control, such as a disturbance observer method [6], robust right co-prime factorization and pre-compensator [7] have been proposed for controlling thermal plate system. In general, robust control is designed based on plant parameters and uncertain parameters for synthesizing robust controller to deal with disturbance signal and parameters varying of plant model.

Model predictive control (MPC) is a modern powerful control strategy in industry and process control. MPC is a form of control in which the current control action is obtained by solving a finite-time constrained optimization online to minimize future tracking error. In [9] model predictive control has been proposed in thermal plate water cooler for saving energy consumption by optimal cost function of control effort.

In this paper, temperature of cold side on thermoelectric plate has been controlled by using model predictive control with constraints on a control input, control signal rate and fin temperature of a heat sink is estimated by using Kalman filter. The proposed strategy is verified in both simulation and experiment tests.

A comparison between model predictive control with constraints and model predictive control without constraints, based on tracking performance of cold side temperature and the variation of fin temperature, is presented.

The paper is organized as follows. Firstly, the mathematic model based on thermoelectric device with heat sink and AC fan system is described. Secondly, the hardware of the test system is set up. Thirdly, the system identifier for finding plant parameters by least square method is presented in this section. Next, model predictive control without constraints model predictive control with constraints on a control input, control signal rate and fin temperature are described in this section. Further, experiment result is given to confirm the proposed method. Finally, the conclusion of this work is presented in the final section.

2. Mathematic model

This work use conservation of energy [1] for developing a system model of thermoelectric with fan as shown in Fig 2.

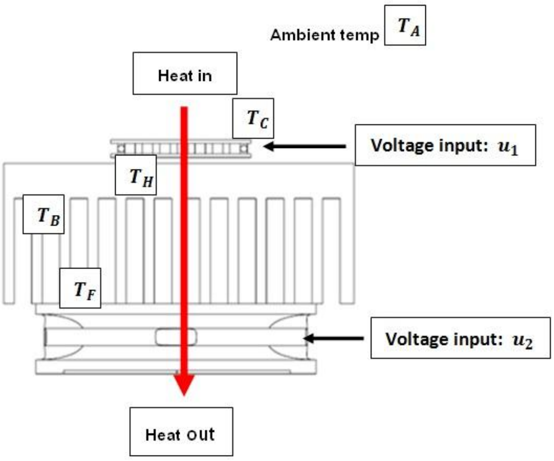

Fig. 2 diagram of thermoelectric with fan system

Where:

T_A is an ambient temperature (°C)

T_C is cold side temperature (°C)

T_H is hot side temperature (°C)

T_B is base of heat sink temperature (°C)

T_F is fin of heat sink temperature (°C)

u_1 is a control input for controlling cold side of thermoelectric plate temperature (V)

u_2 is a control input for controlling AC fan speed (V)

From Fig. 2, the mathematic model of cold side plate, hot side plate, base of heat sink and fin of a heat sink can be found by conservation of energy as shown in Eq. (1) - (4), respectively.

$$\frac{dT_c}{dt} = -\frac{1}{R_{h1}C_c}T_C + \frac{1}{R_{h1}C_c}T_A + \frac{1}{R_{TEC}\,C_c}(T_H - T_C) + \frac{1}{R_{TEC}\,C_c}u_1 - \frac{1}{R_{FAN}\,C_c}u_2 \quad (1)$$

$$\frac{dT_H}{dt} = -\frac{1}{R_{h1}C_H}T_H + \frac{1}{R_{h1}C_H}T_A - \frac{1}{R_{TEC}\,C_H}(T_H - T_C) + \frac{1}{R_{TEC}\,C_H}u_1 - \frac{1}{R_{FAN}\,C_H}u_2 \quad (2)$$

$$\frac{dT_B}{dt} = \frac{1}{R_{TEC}\,C_B}(T_C - T_H) + \frac{1}{R_B C_B}(T_H - T_B) - \frac{1}{R_{hB}C_B}(T_B - T_A) - \frac{1}{R_F C_B}(T_B - T_F) + \frac{1}{R_{TEC}\,C_B}u_1 - \frac{1}{R_{FAN}\,C_B}u_2 \quad (3)$$

$$\frac{dT_F}{dt} = \frac{1}{R_B C_F}(T_H - T_B) + \frac{1}{R_F C_F}(T_B - T_F) - \frac{1}{R_F C_F}(T_F - T_A) + \frac{1}{R_{TEC}\,C_F}u_1 - \frac{1}{R_{FAN}\,C_F}u_2 \quad (4)$$

Where:

R_{h1} is thermal resistant of ambient temperature $(°\mathrm{C}/W)$.

C_i is thermal capacitance of substrate surface and subscripts i is represented C, H, B and F for a cold side, hot side, base of heat sink and heat sink temperature at fin location, respectively.

R_i is thermal resistant of substrate surface and subscripts i is represented C, H, B and F for a cold side, hot side, base of heat sink and heat sink temperature at fin location, respectively.

R_{TEC} is thermal resistant of thermoelectric plate $(°\mathrm{C}/W)$.

R_{FAN} is a time constant for reducing accumulated heat energy in a heat sink.

The state-space representation of the plant can be shown in Eq. (5).

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \quad (5)$$

Where the state - space vectors are $x(k) = [T_C(k)\ T_H(k)\ T_B(k)\ \ T_F(k)\ ]^T$ and control input vectors are $u(k) = [u_1(k)\ \ u_2(k)]^T$. The system matrix can be shown in equation (6).

$$A = \begin{bmatrix} a_{C11} & a_{H12} & 0 & 0 \\ a_{C21} & a_{H22} & 0 & 0 \\ a_{C31} & a_{H32} & a_{B33} & a_{F34} \\ 0 & a_{H42} & a_{B43} & a_{F44} \end{bmatrix} \tag{6}$$

The variables in a system matrix (6) can be expanded as shown in Eq. (7) - (17).

$$a_{C11} = \frac{-R_{TEC}C_C - R_{h1}C_C}{R_{h1}C_C R_{TEC}C_C} \tag{7}$$

$$a_{C21} = \frac{1}{R_{TEC}C_H} \tag{8}$$

$$a_{C31} = \frac{1}{R_{TEC}C_B} \tag{9}$$

$$a_{H12} = \frac{1}{R_{TEC}C_C} \tag{10}$$

$$a_{H22} = \frac{-R_{TEC}C_H - R_{h1}C_H}{R_{h1}C_H R_{TEC}C_H} \tag{11}$$

$$a_{H32} = \frac{-R_B C_B + R_{TEC}C_B}{R_{TEC}C_B R_B C_B} \tag{12}$$

$$a_{H42} = \frac{1}{R_B C_F} \tag{13}$$

$$a_{B33} = \frac{-R_{hB}C_B - R_B C_B - R_F C_B}{R_B C_B R_{hB} C_B R_F C_B} \tag{14}$$

$$a_{B43} = \frac{-R_F C_F + R_B C_F}{R_B C_F R_F C_F} \tag{15}$$

$$a_{F34} = \frac{1}{R_F C_B} \tag{16}$$

$$a_{F44} = \frac{-2}{R_F C_F} \tag{17}$$

The control input matrix can be shown in equation (18).

$$B = \begin{bmatrix} a_{uc1} & a_{uc2} \\ a_{uh1} & a_{uh2} \\ a_{ub1} & a_{ub2} \\ a_{uf1} & a_{uf2} \end{bmatrix} \tag{18}$$

Where:

$a_{uc1} = 1/R_{TEC}C_c\,, a_{uc2} = -1/R_{FAN}C_c$
$a_{uh1} = 1/R_{TEC}C_H\,, a_{uh2} = -1/R_{FAN}C_H$
$a_{ub1} = 1/R_{TEC}C_B\,, a_{ub2} = -1/R_{FAN}C_B$
$a_{uf1} = 1/R_{TEC}C_F\,, a_{uf2} = -1/R_{FAN}C_F$

The essential parameters of a thermoelectric plate can be extracted from open-loop experimental by system identification such as, least square algorithm at working condition.

3. Hardware setup

This work use numerous equipments for setting the experiment such as, power device used to distribute 12 Volt DC from power supply to a thermoelectric plate when control signal from data acquisition (Arduino MEGA 2560) is applied to power device in a finite time, solid state relay used to distribute one phase electrical power to AC similarly to power device. Thermocouple type K use for measuring the dynamic of temperatures at difference location such as, on the cold side of a plate surface, hot side of thermoelectric plate, base of heat sink and tip location at fin of a heat sink. This work use IC AD595 for amplifying the temperature signal from thermocouple and low-pass filter for reducing the amplitude of a signal. The amplified of a temperature signal is used to feedback data for calculating of an error signal in the controller section. In this study MATLAB and Simulink software used to written the algorithm program of open-loop step test for finding system parameters and written the different control algorithms, for example, model predictive control with and without constraints for studying the ability of controllers in thermoelectric plate system. The block diagram of each equipment can be seen in Fig. 3.

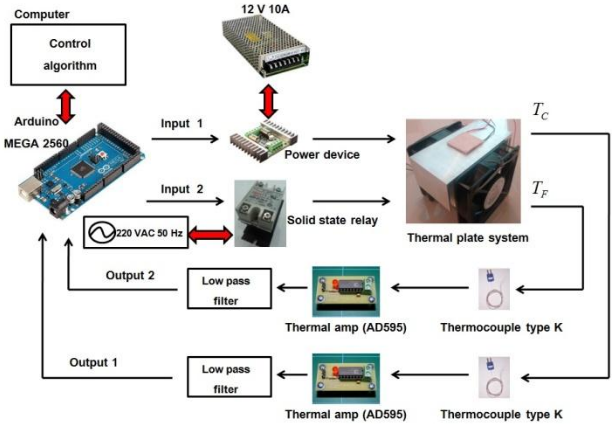

Fig. 3 Block diagram of hardware and software configuration.

This work use standard size of thermoelectric module as a demo of the testing system. The standard thermoelectric is 40 Watt 40 mm x 40 mm x 5 mm in width, length and thin, respectively. The size of heat sink is 70 mm in width 150 mm in length and the length of fin is 50 mm. The size of heat sink is enough for removing an accumulated heat from the hot size of thermoelectric according to [12].

The idle of optimum size of a thermoelectric can be seen in reference [13] for finding the optimum size with dimensional analysis.

4. System identification

Essential parameters of the system can be determined by least square algorithm [10], which is a process to find unknown parameters from a physical model, the algorithm can be written as

$$Y = \Phi\theta \tag{19}$$

Where:

θ is an unknown parameter vector, such as a model parameter.

Φ is known regression matrix, which is the matrix contain the data of state and input, from experiment condition.

Y is known measurements vector, such as output of the system.

The solution of θ can be found from Eq.(20).

$$\theta = \Phi^{-1}Y \tag{20}$$

The solution of least square θ_{LS} can be written as

$$\theta_{LS} = (\Phi^T\Phi)^{-1}\Phi^T Y. \tag{21}$$

Mathematic model of cold side, hot side based location and fin temperature from the system matrix can be rewritten in the form of least square algorithm as shown in Eq.(22) - (25), respectively.

$$\underbrace{\frac{T_{C_{k+1}}-T_{C_k}}{T_s}}_{Y} = \underbrace{[T_C\ T_H\ T_B\ \ T_F\ u_1\ u_2]}_{\Phi}\underbrace{\begin{bmatrix}a_{C11}\\a_{H12}\\0\\0\\a_{uc1}\\a_{uc2}\end{bmatrix}}_{\theta} \tag{22}$$

$$\underbrace{\frac{T_{H_{k+1}}-T_{H_k}}{T_s}}_{Y} = \underbrace{[T_C\ T_H\ T_B\ \ T_F\ u_1\ u_2]}_{\Phi}\underbrace{\begin{bmatrix}a_{C21}\\a_{H22}\\0\\0\\a_{uh1}\\a_{uh2}\end{bmatrix}}_{\theta} \tag{23}$$

$$\underbrace{\frac{T_{B_{k+1}}-T_{B_k}}{T_s}}_{Y} = \underbrace{[T_C\ T_H\ T_B\ T_F\ u_1\ u_2]}_{\Phi}\underbrace{\begin{bmatrix}a_{C31}\\ a_{H32}\\ a_{B33}\\ a_{F34}\\ a_{ub1}\\ a_{ub2}\end{bmatrix}}_{\theta} \qquad (24)$$

$$\underbrace{\frac{T_{F_{k+1}}-T_{F_k}}{T_s}}_{Y} = \underbrace{[T_C\ T_H\ T_B\ T_F\ u_1\ u_2]}_{\Phi}\underbrace{\begin{bmatrix}0\\ a_{H42}\\ a_{B43}\\ a_{F44}\\ a_{uf1}\\ a_{uf2}\end{bmatrix}}_{\theta} \qquad (25)$$

Where $T_s = 0.05s$ is the sampling time for logging data. The open-loop test is used to identify essential parameters. The identifier process uses input signal condition as a three step down signal for studying the dynamic of temperature below ambient air on a cold side of thermoelectric plate.

The input condition signal of Ac fan is applied to a constant value for removing accumulated heat from a heat sink as a constant rate.

The input conditions of thermoelectric plate and AC fan have been recorded from logging the process every sampling time.

The dynamic respond of cold side, hot side, based of heat sink and fin of heat sink temperature have been measured by thermocouples type K for a set of output data for using in an identification process.

Plant parameters can be extracted by identification process from recorded input-output data, so the system matrix and input matrix of thermoelectric plate with fan system can be written as Eq.(26),(27), respectively.

$$A = \begin{bmatrix}0.843 & -0.001 & 0 & 0\\ -0.05 & 0.772 & 0 & 0\\ -0.03 & 0.1077 & 0.67 & 0.066\\ 0 & 0.1077 & -0.33 & 0.987\end{bmatrix} \qquad (26)$$

$$B = \begin{bmatrix}0.0853 & -0.0096\\ 0.0277 & 0.0033\\ 0.0022 & 0.0156\\ 0.0021 & 0.0147\end{bmatrix} \qquad (27)$$

The output matrix set as Eq.(28) for studying only cold side and fin of heat sink temperature.

$$C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (28)$$

4.1 Validation of plant model

The percentage of best fit criterion of the model is used to check an accurate of model from system identification method. This work use best fit criterion according to [10] as shown in Eq. (29).

$$Best\ Fit = \left(1-\frac{\sum_{i=1}^{N}|y_i-\hat{y}_i|}{\sum_{i=1}^{N}|y_i-\bar{y}_i|}\right)\times 100\% \qquad (29)$$

Where N is the total number of data, y_i is a measurement output at i time step and $\hat{y}_i$ is a simulation output from the model at i time step.

The best fit of each parameters are 92.7 % for T_C parameter, 93.8% for T_H parameter, 89.8 % for T_B parameter and 84.4% for T_F parameter.

The best fit criterion for each parameter in thermoelectric with fan can be shown in Fig.4.

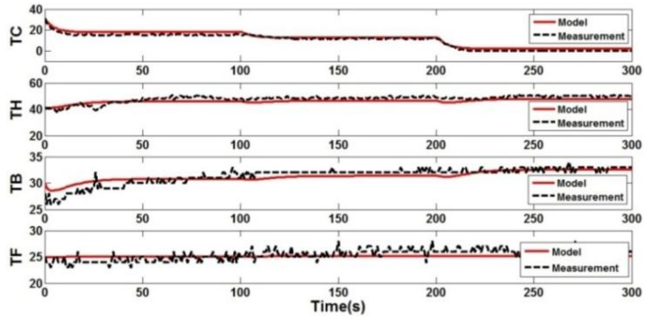

Fig.4 The comparison of the measurement signal (dash line) and simulated output generated by model (solid line) between, case (a) T_C temperature and model, case (b) T_H temperature and model, case (c) T_B temperature and model and case(d) T_F temperature and model.

5. Controller design

In this work, model predictive with and without constraints are used to control cold side temperature and fin of heat sink temperature for studying performance of difference controllers.

The general design objective of model predictive control is to compute a trajectory of future manipulated variables u_1 and u_1 to optimize the future behavior of system outputs T_C and T_F. The optimization is performed within a limited time window by giving plant information at the start of the time window [11].

This work use model predictive control MATLAB toolbox as shown in Fig. 5 [5], [8] for designing model predictive with and without constraints.

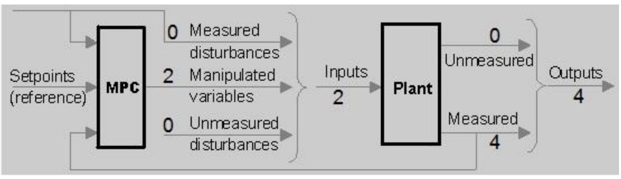

Fig. 5. MATLAB toolbox for designing model predictive control [5].

The system matrix A, control input matrix B and output matrix C as shown in Eq. (26) - (28), respectively, have been imported into MPC toolbox in MATLAB program for designing model predictive controller with and without constraints.

In the case of constraints, this work set numerous constraints, such as level of control input to thermoelectric, rate of change in control input to thermoelectric and upper and lower bound of fin temperature as shown in Eq.(29) - (31), respectively.

$$0V \leq u_1 \leq 3V \tag{29}$$

$$-3V \leq \Delta u_1 \leq 3V \tag{30}$$

$$35°\mathrm{C} \leq T_F \leq 40°\mathrm{C} \tag{31}$$

The designed controllers from MPC toolbox can export to simulink program for implementation in real-time system by using Arduino MEGA 2560.

5.1 Fin temperature estimate

In the design of model predictive controllers, this work assumed the state-vector information $x(k)$ are measurable at time step k [11].

In reality, not all state variables are measured or available. Some of them may be impossible to measure because specification of hardware or size of the sensor for mounting at the surface hardware.

In model predictive control, all state must available, but some states are not.

Therefore, this work is to study the estimation process of state when assumed fin temperature cannot measure and develop the kalman filter to estimate unknown state variable based on process measurement and mathematic model of thermoelectric system with fan.

The kalman filter is constructed using Eq. (32) [11].

$$\hat{x}(k+1) = \underbrace{A\hat{x}(k) + Bu(k)}_{model} + \underbrace{K_{ob}\big(y(k) - C\hat{x}(k)\big)}_{correction\ \ term} \quad (32)$$

Where optimal kalman filter gain K_{ob} is solved recursively for $i = 0,1,\dots,n$ by using Eq. (33).

$$K_{ob}(i) = AP(i)C^T(\Gamma + CP(i)C^T)^{-1} \quad (33)$$

The matrix P can be calculated as shown in Eq. (34).

$$P(i+1) = A\big(P(i) - P(i)C^T \times$$
$$\Gamma + CPiCT - 1CPiAT + \Theta \quad (34)$$

The kalman filter consists of two terms. The first term is the original model and the second term is a correction term based on the error between the measured output and predicted output using the estimate $\hat{x}(k)$ [11].

6. Experimental study

This work use five reference signal in a cold side on thermoelectric and set the reference signal of fin temperature to 37 ℃. Cold side, fin and estimate of fin temperature are shown in Fig. 6 - 8.

From Fig. 6 (a), The respond of cold side temperature of thermoelectric plate when using model predictive control with constraints as shown in Eq.(29) - (30) is better than model predictive control without constraints in term of transient respond.

In the first 50 second of a line graph from Fig. 6(a), There is a sharp drop in temperature from around 42 ℃ to below 20 ℃ followed by a small increase in temperature to the set point signal at 20 ℃. The respond of temperature when using MPC with constraints (solid-line) as a controller is faster recovery to set point signal then using MPC as a controller.

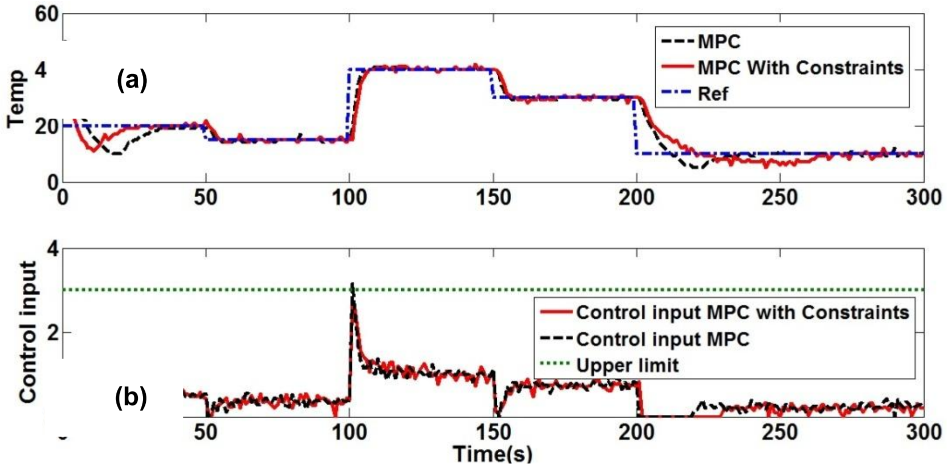

Fig.6. Case (a), Experiment result of MPC (dash-line), MPC with constraints (solid-line), Reference signal of cold side temperature (dash dot-line). Case (b), Control input to thermoelectric plate where (dash-line) is MPC, (solid-line) is MPC with constraints, (dot-line) is an upper limit of control input.

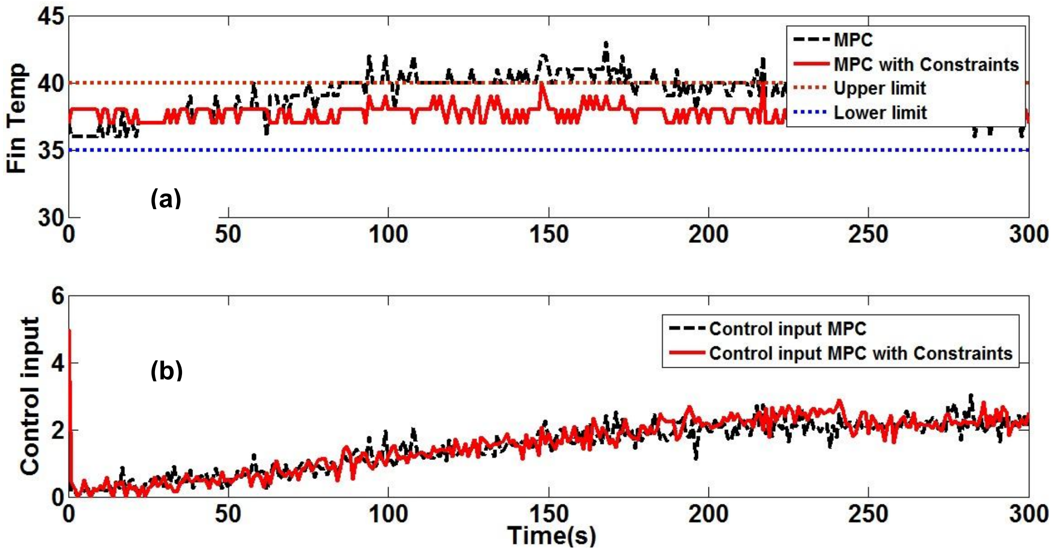

Fig.7. Case (a), Fin temperature of MPC (dash-line), MPC with constraints (solid-line), upper and lower of fin temperature (dot-line). Case (b), Control input to thermoelectric plate where (dash-line) is MPC, (solid-line) is MPC with constraints.

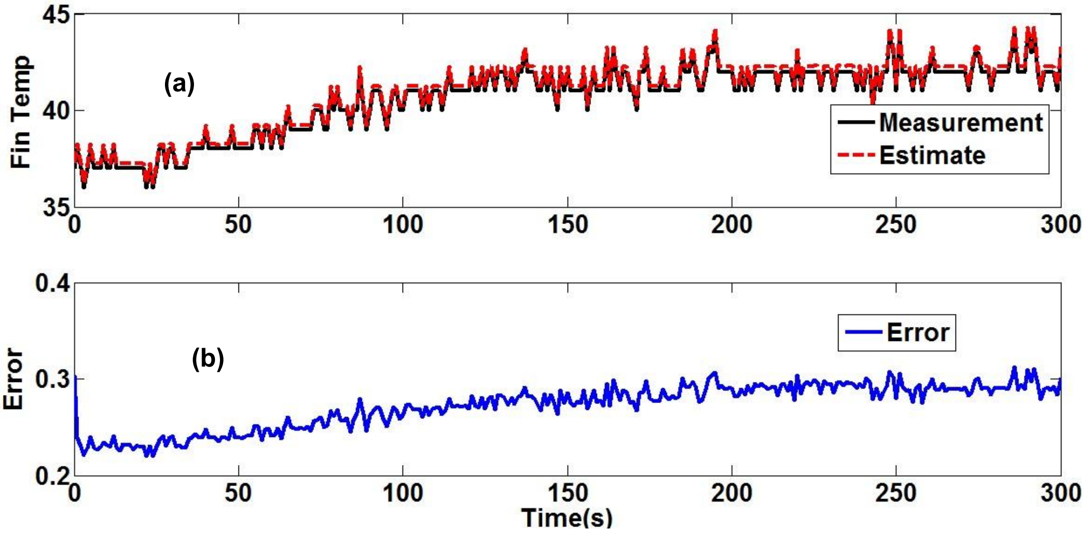

Fig. 8. Case (a), Estimation of fin tempereture using kalman filter (dash-line) comparison with measured signal (solid-line). Case (b), Error between estimation and measurement.

From 50 to 200 second, the performance of two controllers are same by kept the temperature of thermoelectric plate to set point signal in term of transient and steady-state.

From the last 100 second, the temperature when using MPC with constraints as a controller decreased significantly and reached the set point signal at 10 ℃ better than temperature when using MPC without constraints as a controller.

The control input to thermoelectric plate (u_1) exceed the constraint limit in the case of using MPC as a controller as shown in Fig. 6 (b).

Therefore, the set of constraints form Eq. (29) and (30) have ability to limit the voltage source for supplying the thermoelectric and bring the respond temperature to a set point signal better than does not using constraints in MPC controller.

From Fig. 7(a), the respond of fin temperature when using MPC as a controller is exceed an upper limit of constraints in Eq. (31).

The fin temperature when using MPC with constraint is fluctuates significantly between 36℃ to 40℃ and does not out of limit constraint as shown in Fig. 7(a).

The control input to AC fan when using model predictive and model predictive control with constraints are same in tend of signal, but the magnitude of oscillate control input in the case with constraints is less than the case without constraints as shown in Fig 7(b).

From Fig. 8, Kalman filter can estimate fin temperature with a maximum error between estimation and measurement signal is 0.28 as shown in Fig 8(b).

7. Conclusion

Experimental results show that the performance of proposed controller is better than MPC in all of operating condition for controlling on the cold side of thermoelectric plate and fin temperature.

The MPC without constraints using an unnecessary control input signal, so the temperature respond of thermoelectric and fin of heat sink are worst than temperature respond when using MPC with constraints as a controller.

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8. Nomenclature

T_A	ambient temperature $^{\circ}\mathrm{C}$.
T_C	cold side temperature of thermoelectric plate $^{\circ}\mathrm{C}$.
T_H	hot side temperature of thermoelectric plate $^{\circ}\mathrm{C}$.
T_B	base of heat sink temperature $^{\circ}\mathrm{C}$.
T_F	fin of heat sink temperature $^{\circ}\mathrm{C}$.
u_1	control input for controlling cold side of thermoelectric temperature V.
Δu_1	control input rate for controlling cold side of thermoelectric temperature V.
u_2	control input for controlling AC fan speed V.
R_{h1}	thermal resistant of ambient temperature $^{\circ}\mathrm{C}/W$.
R_C	thermal resistant of cold side of thermoelectric plate $^{\circ}\mathrm{C}/W$.
R_H	thermal resistant of hot side of thermoelectric plate $^{\circ}\mathrm{C}/W$.
R_B	thermal resistant of base of heat sink $^{\circ}\mathrm{C}/W$.
R_F	thermal resistant of fin of heat sink $^{\circ}\mathrm{C}/W$.
R_{TEC}	thermal resistant of thermoelectric plate $^{\circ}\mathrm{C}/W$.
R_{FAN}	time constant for reducing accumulated heat energy from a heat sink.
C_C	thermal capacitance of cold side of thermoelectric plate $j/kg\cdot{}^{\circ}\mathrm{C}$..

C_H	thermal capacitance of hot side of thermoelectric plate $j/kg \cdot {}^{\circ}\mathrm{C}$.
C_B	thermal capacitance of base of heat sink $j/kg \cdot {}^{\circ}\mathrm{C}$.
C_F	thermal capacitance of fin of heat sink $j/kg \cdot {}^{\circ}\mathrm{C}$.
A	system matrix (see Eq. (6)).
$x(k)$	state-space vectors.
$\hat{x}(k)$	estimate of state-space vectors.
$u(k)$	control input vectors.
B	input matrix (see Eq. (18)).
C	output matrix (see Eq. (28)).
Y	known measurement vector.
K_{ob}	optimal kalman filter gain.
P	covariance matrix of the state estimate.

Greek symbols

Φ	known regression matrix.
θ	unknown parameters vector.
θ_{LS}	solution of least square (see Eq. (21)).
Γ	covariance matrix of disturbances.
Γ	covariance matrix of disturbances.
Θ	covariance matrix of noise.

Subscripts

A	ambient air.
C	cold side of the thermoelectric plate.
H	hot side of the thermoelectric plate.
B	base of heat sink.
F	fin of heat sink.
$1,2$	number of control input
h_1	convection coefficient.
TEC	thermoelectric cooler.
FAN	AC fan.
LS	least square.
k	time step.