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Model Reference Input Shaping Using Quantitative Feedback Control

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Abstract

Input shaping technique convolutes reference signal with a sequence of impulses. The amplitudes and time locations of impulses are designed such that the resulting shaped reference signal avoids exciting lightly-damped modes of the system. This results in zero residual vibration for quick point-to-point movement. However, the input shaping technique is sensitive to accuracy of the mode parameters (natural frequency and damping ratio) of the system. Adding more impulses to the sequence makes it more robust against parameter uncertainty at the expense of slower shaped reference signal. In this paper, for the first time, a quantitative feedback control is used in the loop to make the closed-loop system match a known reference model, a so-called model matching. There are two benefits in doing this. First, the input shaper sees the closed-loop system as the reference model whose mode parameters are known perfectly. Therefore, the input shaper need not be robust, resulting in fewer impulses usage and faster shaped reference signal. Second, the quantitative feedback control can reduce vibration from other sources such as plant-input, plant-output disturbances, and noise. Simulation shows that the proposed technique can withstand large plant uncertainty with fast move time when compared to traditional robust input shaper.

***Keywords**:* Closed loop; Robust input shaping; Quantitative feedback; Vibration reduction; Flexible joint; Model reference.

1. Introduction

The input shaper is viewed as a filter that reduces the vibrations induced by external sources such as reference input, disturbances, and noise. Since vibration reduction is not the only objective of the control system, the input shaper is often implemented together with feedback controller.

In general, there are two configurations; either the input shaper is placed outside or inside the feedback loop. Although placing the input shaper inside the loop can reduce vibrations from all external sources, there are some limitations for the design of the feedback controller due to additional time delay introduced by the input shaper. Meanwhile, placing the input shaper outside the loop only reduces vibration from the reference input but without additional limitations.

Several researchers have investigated placing the input shaper outside the loop together with different types of feedback controller: iterative learning controller by Zolfagharian et al. [1]; genetic algorithm by Aldebrez et al. [2]; sliding mode controller by Pai [3] and Hu et al. [4]; PD controller by Huey and Singhose [5] and Kenison and Singhose [6]; and adaptive controller by Dharne and Jayasuriya [7]. A feedback controller based on quantitative feedback theory (QFT) is used in this paper.

QFT was introduced by Horowitz [8] as a loop shaping control design technique that extends the work of Bode [9]. It is so-called quantitative because various specifications such as tracking, disturbances and noise rejections, relative stability, and model matching are graphically displayed on the Nichols chart for the designer to make trade-off among them. At a specific frequency, instead of being viewed as a point on the Nichols chart, the plant is now viewed as a template, which is an area containing possible plant values due to uncertainties. The loop shaping process results in a feedback controller and a prefilter that ensure the closed-loop system meet all specifications for all uncertain plants. Model matching capability of the QFT is what emphasized in this paper.

Several researchers have used feedback control system to match the closed-loop system to a reference model. The input shaper is then designed using vibratory mode parameters of the reference model. Pai [3] used ADALINE neural network to enforce both sliding and reaching phases in a sliding mode control to make model matching error go to zero despite parameter variations and external disturbance. Only simulation was presented, and the complexity and global convergence of this time-domain algorithm may pose implementation problem. Yu and Chang [10] used a PI controller to match the closed-loop system of a piezoelectric nano-positioner to a reference model, while Yu and Chang [11] used a P controller to match the closed-loop system of a dual solenoid actuator to a reference model. Both works used zero-vibration input shaper. However, the design of the PI or P controller was heuristic; and there was no mention about attainable specifications or allowable uncertainty. Dharne and Jayasuriya [7] used a direct model reference adaptive control to match the closed-loop system to a reference model. The technique may therefore be susceptible to insufficiently persistent excitation of input, global instability, and divergence of adapted parameters.

In this paper, instead of using signal-based reference model matching as in other previous works, system-based method is considered. Closeness between the reference model and the closed-loop system from reference input to output is formulated as a frequency-domain specification. The plant is allowed to be uncertain. This uncertainty together with the specification can be converted to bounds on the Nichols chart. A feedback controller, is designed to loop-shape the open-loop system to satisfy all bounds. Simulation on a benchmark mass-damper-spring plant, with 35% uncertainties in all plant parameters, shows that the closed-loop system matches the reference model well within the pre-specified specification. A zero-vibration (ZV) input shaper is placed outside the loop and is designed from the vibratory mode parameters of the reference model. The responses to step reference inputs, for all uncertain plants, show significant vibration reduction using the proposed system compared to using the open-loop ZV and robust input shapers. Vibrations induced by disturbances and noise are also reduced by the system.

This paper is organized in this way. Section 2 contains details of robust input shaping. Details include finding amplitudes and time locations of the impulses in the input shaper, how robust input shaper is obtained, and how to apply the input shaper as an open-loop system. Section 3 contains the proposed model reference input shaping system, consisting of a ZV input shaper and a quantitative feedback controller. In this section, model matching bounds are derived. Section 4 presents a simulation on a benchmark mass-damper-spring plant. The control design steps are demonstrated, followed by the simulation results. Conclusions are given in Section 5.

2. Robust Input Shaping

Input shaping was originated by Singer and Seering [12] based on the posicast control of Smith [13]. Excellent review of the input shaping technique can be found in Singhose [14].

The technique includes finding amplitudes and time locations of the impulses in an impulse train by solving: (1) residual vibration constraint; (2) robustness constraint; (3) impulse amplitude constraint; and (4) time optimality constraint. The resulting impulse train is then convoluted with the reference input resulting in a shaped reference input that 1) avoids exciting the system vibratory modes for minimal residual vibration; 2) is robust against vibratory mode parameter variations; 3) has the same final value as that of the original reference input; and 4) arrives at the final value in the shortest time.

By solving constraints (1), (3), and (4), one obtains a so-called *zero vibration (ZV)* input shaping with two impulses whose magnitudes and time locations are given by

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \dfrac{1}{1+K} & \dfrac{K}{1+K} \\ 0 & \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} \end{bmatrix}, \qquad (1)$$

where

$$K = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}.$$

In addition, if the robustness constraint (2) is also included, one obtains a so-called *zero vibration derivative (ZVD)* input shaping with three impulses whose magnitudes and time locations are given by

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \dfrac{1}{1+2K+K^2} & \dfrac{K}{1+2K+K^2} & \dfrac{K^2}{1+2K+K^2} \\ 0 & \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} & \dfrac{2\pi}{\omega_n\sqrt{1-\zeta^2}} \end{bmatrix}. \qquad (2)$$

If the robustness constraint also includes second-order derivatives onward, the resulting input shapers will be with four or more impulses and are so-called *ZVDD, ZVDDD,* and so on. More impulses means more robustness but with longer time to the final reference value.

Fig. 1 shows the input shaper *IS* applied to an uncertain plant *P*, where $\bar{r}$ is the original reference input, r is the shaped reference input, and y is the output.

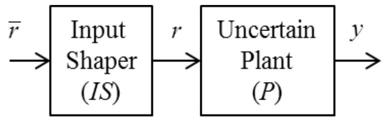

Fig. 1 Open-loop application of the input shaper to an uncertain plant.

3. Reference Input Shaping with Quantitative Feedback Controller

Even though the robust input shapers, such as the ZVD, can handle uncertainties in parameter variations, they come at a price of having longer time to reach final reference set-point. The shaped reference input of the ZVD reaches its final value at $t_2 = 2\pi / \left[\omega_n \sqrt{1-\zeta^2} \right]$, which is twice that of the ZV.

For shorter move time, in this section, we investigate using feedback control system together with the non-robust ZV input shaper. The job of handling the parameter uncertainties is delegated to the feedback control system. The input shaper is placed outside of the loop to reduce vibration induced by reference input, while the feedback system makes the uncertain closed-loop system match a reference model and reduces vibration induced by disturbances and noise.

Consider a closed-loop system in Fig. 2, whose, in addition to those in Fig. 1, e, d_I, u, d_O, and n are tracking error, plant-input disturbance, control input, plant-output disturbance, and noise, respectively. The quantitative feedback control system consists of a feedback controller G and a prefilter F. They will be designed so that the mapping from *r* to *y* matches a reference model *M*, that is,

$$\left| \frac{Y}{R} - M \right| = \left| \frac{PGF}{(1+PG)} - M \right| < \delta_m,$$

where δ_m is a small number that determines how close the uncertain closed-loop system be to the exact reference model. Compared to that in Fig. 1, the quantitative feedback control system *G* and *F* helps the input shaper in handling the uncertainty from the plant.

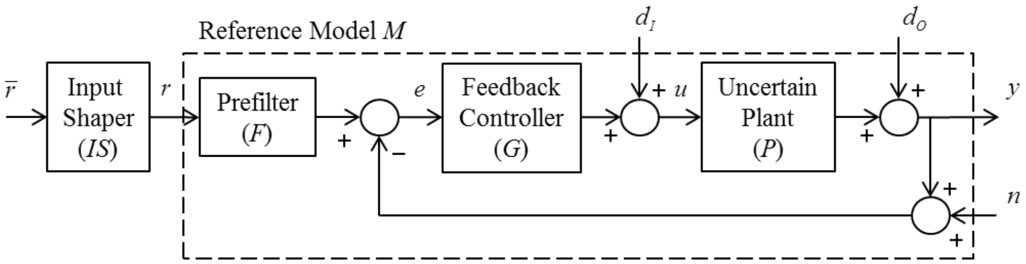

Fig. 2 Model reference input shaping with quantitative feedback controller.

When $F = M$ and $M = P_0$, where P_0 is the nominal plant, the frequency-domain specification above becomes

$$\left| \frac{P_0}{1+PG} \right| < \delta_m. \tag{3}$$

Further derivation leads to $|G + 1/P| > \delta_m^{-1} |P_0 / P|$, which shows that, on the complex plane, *G* must be shaped so that $|G+1/P|$ is greater than $\delta_m^{-1} |P_0 / P|$ as shown in Fig. 3.

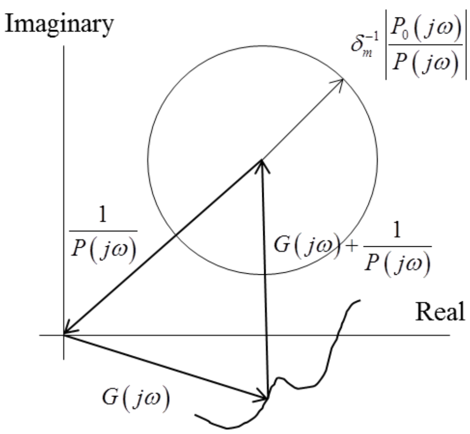

Fig. 3 Allowable region for $G(j\omega)$.

4. A Simulation Example

An *m-c-k* benchmark problem, shown in Fig. 4, will be used as the uncertain plant to compare the system in Fig. 2 to that in Fig. 1. This benchmark problem was used in Cole and Wongratanaphisan [15]. The transfer function from x_1 to x_2 is given by

$$\frac{X_2}{X_1} = \frac{cs+k}{ms^2+cs+k},$$

whose nominal parameter values are $m = 1\,kg$, $k = 1.5791\times10^4\ kg.s^{-2}$, and $c = 25.1327\ kg.s^{-1}$, corresponding to a natural frequency $\omega_n = 40\pi\ rad/s$ and a damping ratio $\zeta = 0.1$.

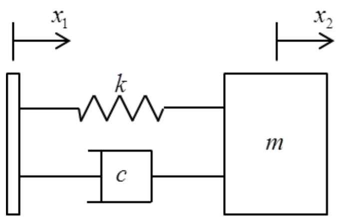

Fig. 4 A benchmark *m-c-k* system.

Assume 35% uncertainties in each parameter, that is, $m \in \{0.65, 1.35\}$, $c \in \{16.336, 33.929\}$, and $k \in \{1.0264, 2.1318\}\times10^4$, and select working frequencies $\omega = \{10, 50, 100, 120, 150, 200, 500\}\ rad.s^{-1}$, which sufficiently cover low, high, as well as the natural frequency of the system. Uncertain plant regions, so-called *plant templates*, are plotted on the Nichols chart. Their shapes are shown in Fig. 5, in which the asterisks mark nominal plants. The control system will be designed with specification (3) for all uncertain plants in the uncertain regions.

For the feedback system in Fig. 2, the specification (3) with *G* = 1 and $\delta_m = -20\ dB = 0.1$ is converted to bounds on the Nichols chart as shown in Fig. 6(Top). Each bound represents boundary of the allowable region, shown in Fig. 3, for $L_0 = GP_0$, for all plant uncertainties and for a specified frequency. Fig. 6(Top) also contains the original shape of $L_0 = GP_0$. To be in the allowable region, L_0 must be shaped to be above or outside its bounds for all working frequencies.

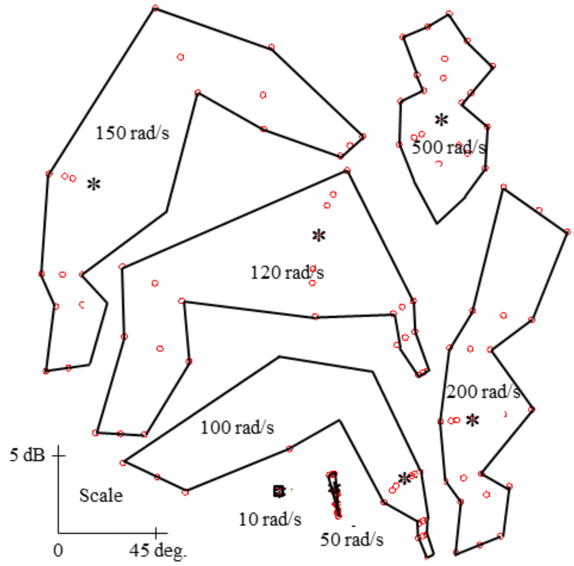

Fig. 5 Plant templates for the m-c-k system.

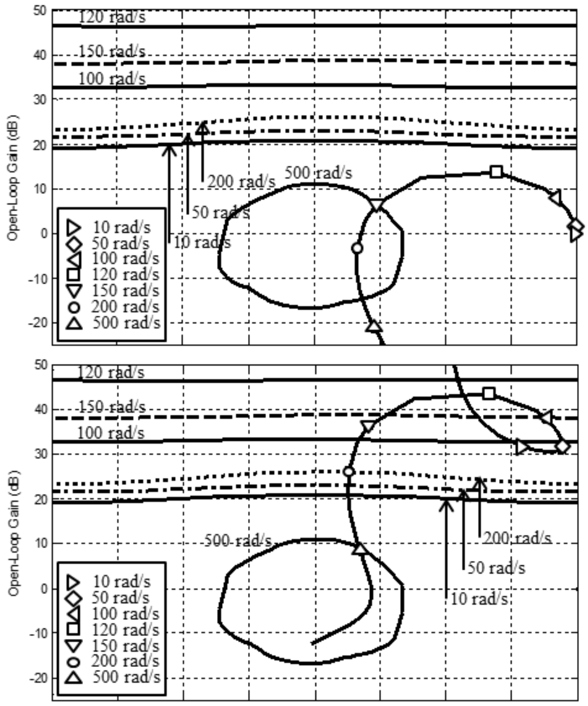

Fig. 6 Loop shaping for the m-c-k system. (Top) Unshaped. (Bottom) Feedback.

Fig. 6(Bottom) contains the shaped L_0. Because the plant is of type 0, to obtain zero step input tracking, we first appended an integrator $1/s$. Then a real zero $(s/7.661+1)$ was added to increase the overall phase of the system. A constant gain of 234.4 was then added to shift the shape up above its bounds. Note that we deliberately violated the bounds at 120 and 150 *rad/s* to avoid too high gains around the natural frequency. Finally, a complex pole $1/\left(s^2/3439^2+2(0.565)s/3439+1\right)$ was added to have higher roll-off rate at high frequencies beyond 1000 *rad/s* to avoid high-frequency noise. The overall feedback controller becomes

$$G=\frac{234.4}{s}\left(\frac{s}{7.661}+1\right)\left(\frac{1}{\frac{s^2}{3439^2}+\frac{2(0.565)s}{3439}+1}\right). \qquad (4)$$

Fig. 7 shows the Bode magnitude plots in the feedback case. Fig. 7(Above) displays the plots of the reference model $M=P_0$ in dashed line and of the closed-loop transfer functions $Y/R=(PGF)/(1+PG)$ in multiple solid lines. Each solid line represents the transfer function computed from each uncertain plant in the set. The feedback controller *G* and the prefilter *F* were designed to make the mapping *Y* / *R* match the reference model *M* according to the specification (3) for working frequencies ranging from 10 to 500 *rad* / *s*. Fig. 7(Below) shows the plots of the reference model $M=P_0$ in dashed line and of the plant *P* in multiple solid lines. It can be seen that without the closed-loop system, the uncertain plant can differ much from its nominal value. Since the input shaper was designed from the reference model $M=P_0$, the deviation should adversely affect its performance in reducing the residual vibration as will be seen next.

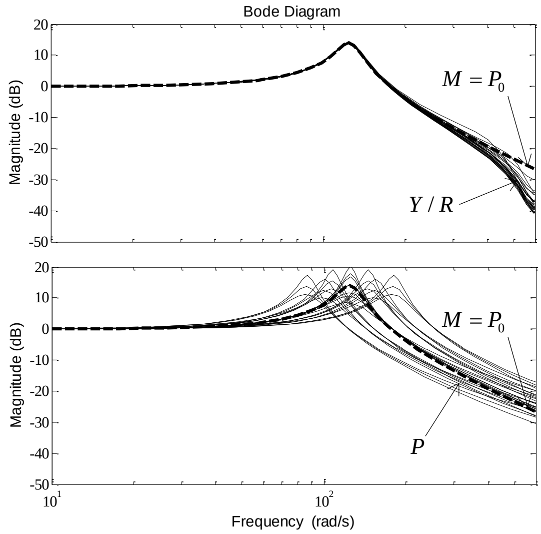

Fig. 7 Bode plots in the feedback case. (Above) The reference model *M* in dashed line and the mapping *Y* / *R*. (Below) The reference model *M* in dashed line and the uncertain plant *P*.

Several time-domain tracking results are presented in Fig. 8 using a sampling period of 0.001 sec. The ZV input shaper is given by (1) and the ZVD input shaper is given by (2). Both were designed from the natural frequency ω_n and the damping ratio ζ of the nominal plant P_0. The model reference input shaping with quantitative feedback control system in Fig. 2 uses the ZV input shaper, the controller *G* given by (4), and the prefilter $F=P_0$.

Fig. 8(a) contains the output x_2 from all uncertain plants for the feedback system in Fig. 2. The shaped reference input from the ZV input shaper is in dashed line. The system delivers good tracking result with almost no overshoot and with short settling time.

Fig. 8(b) and (c) contain the output x_2 for the open-loop system in Fig. 1. Fig. 8(b) is when the ZV input shaper is used, and Fig. 8(c) is when the ZVD input shaper is used. Without the closed-loop system, the outputs from the uncertain plants oscillate heavily even when the robust ZVD input shaper is used.

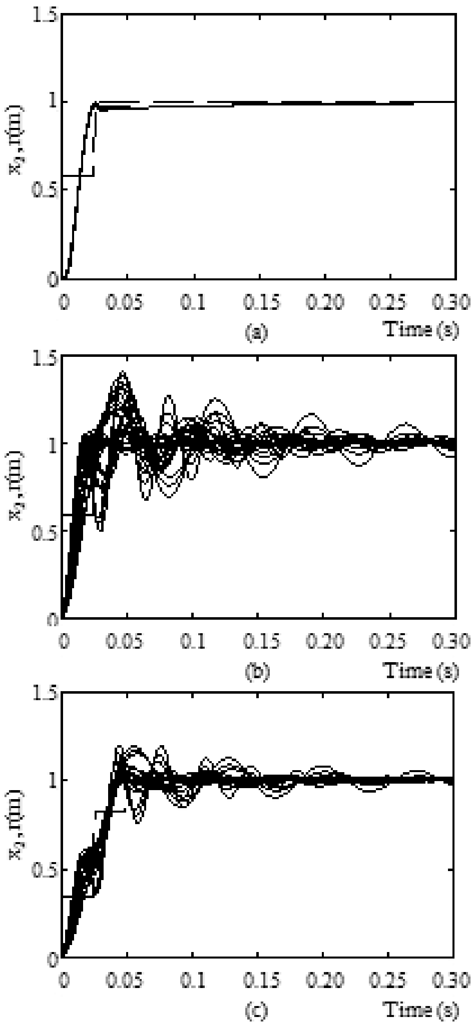

Fig. 8 Tracking of the output x_2: (a) for the feedback system with ZV input shaper, (b) for the open-loop system with ZV input shaper, and (c) for the open-loop system with ZVD input shaper.

Vibration can also come from disturbances and noise. The input shaper, placed outside of the loop, cannot reduce these vibrations. The feedback controller in Fig. 2 can be designed to reject disturbances and noise to suppress these vibrations.

Fig. 9(a) shows the output x_2 from the open-loop system in Fig. 1 when there exists unit-step plant-output disturbance. Fig. 9(b) is the result from the feedback system in Fig. 2. We can see that the feedback system can also reduce vibrations induced by disturbances and noise (not shown in the result) by attenuating their influences on the output.

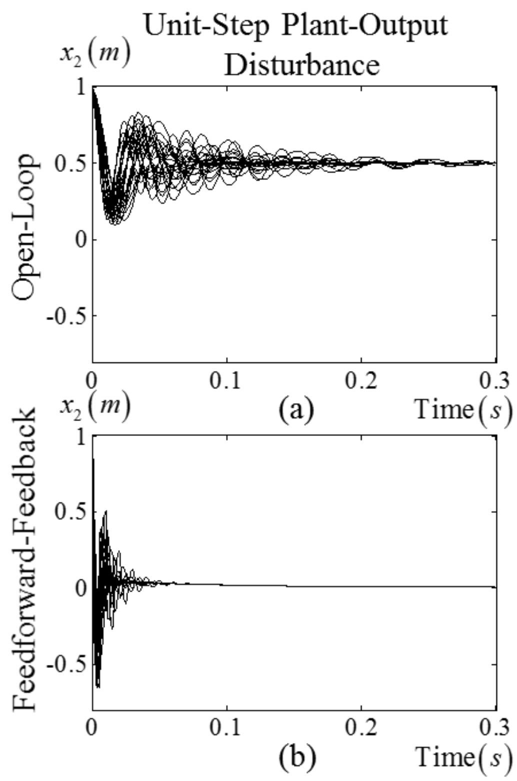

Fig. 9 Output from disturbance. (a) with open-loop system. (b) with feedback system.

5. Conclusions

Two very practical techniques are fused together in this paper. Input shaper reduces vibration from reference input, while quantitative feedback controller reduces vibrations from disturbances and noise. The quantitative controller is also designed to match the closed-loop system to a reference model, whose vibratory mode parameters are used in the design of the input shaper. The quantitative controller explicitly takes into account the amount of the plant uncertainty and various achievable frequency-domain specifications during the design process.

Model matching enhances robustness of the input shaper to plant model uncertainty, without increasing the number of impulse and move time. The quantitative controller can be designed such that the resulting closed-loop system, which matches the reference model, has less and specifiable uncertainty than that of the original uncertain plant.

The proposed technique of using the input shaper and the quantitative controller is simple yet practical enough to be applied to complicated as well as nonlinear plants. For these plants, we need to find a central linear plant model and be sure its uncertain region or plant template covers all possible plants in all operating points. Gain scheduling may be used to obtain multiple linear plant models. This can reduce the size of the uncertain region, resulting in tighter specifications.

6. References

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