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Stabilization of Two-Wheeled Self-Balancing Robot via Immersion and Invariance Method

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Abstract

In this paper, we address nonlinear control via *immersion and invariance (I&I)* method for stabilizing the motion of two-wheeled self-balancing (TWSB) robot where model uncertainties are present. The TWSB robot is a class of *underactuated mechanical systems (UMSs)* by means of it has fewer independent control actuators than degrees of freedom to be controlled. Moreover, by nature of its system structure that has marginal stability, nonlinearity, and non-minimum phase which greatly complicates the control design issue. We use the method of global change of coordinates and *collocated partial feedback linearization (PFL)* to transform the model into a cascade form allowing us to successfully apply the I&I controller. Based on selection of target system that has stable potential energy and positive damping function; a robust I&I controller is designed, which efficiently avoids choosing Lyapunov function. By depends on choosing of the off-manifold variables that satisfy some certain conditions, result in various forms of the I&I control law. The efficiency and effectiveness of the proposed controllers is demonstrated on the TWSB robot with some exogenous output disturbances and variation of model parameters. The stability proof and the simulation results guarantee that all trajectories of the closed-loop systems are asymptotically stable. Moreover, the performance of each designed control law is quite similarly, i.e., the TWSB robot can be stabilized at the upright position while wheels position is brought to the initial state.

***Keywords**:* Nonlinear Systems; Immersion and Invariance; Two-Wheeled Self-Balancing Robot; Underactuated Mechanical Systems; Partial Feedback Linearization

1. Introduction

Two-wheeled self-balancing (TWSB) robot is a pendulum-like on a moving cart, currently used as teaching aids and research experiments. In particular, TWSB robot has wide applications in transportation and industry such as Nbot [2], JOE [9], and Segway [11]. Control of TWSB robot attracted the attention of researchers in the last decade due to their inherent instability, non-minimum phase, nonlinear and complex dynamics. Moreover, it has fewer controls than numbers of degrees of freedom, so-called *underactuated mechanical systems (UMSs)* [6], [8], [16]. In fact, some of degrees of freedom in the system is not excited directly by actuators but controlled through the actuated coordinates.

In order to balance of the TWSB robot is only achieved by considering dynamic equation. An earlier work, TWSB models have been classified into twofold: First, the models based on nonlinear dynamic equations via the Euler-Lagrange (EL) equation, Newton's laws of motion or Kane's method. The natural assumptions are ideal rigid body dynamics, flat and horizontal ground surface, zero wheel slip and no friction. Second, a black-box model of system dynamics [1], [12], [15] such as ARX (auto-regressive with extraneous input), ARMAX (includes moving average), Box-Jenkins, Output-error models and Takagi–Sugeno (TS) fuzzy model [17]. These models can also be experimentally identified the parameters of both linear and nonlinear models.

The simplest model only allows for straight-line (or 1-dimensional) motion is considered. Only motion in the vertical plane (see Fig. 1) is modeled, involving the two degrees of freedom yielding two states for each degree, i.e., longitudinal displacement and tilt of the cart body. In literatures, without trying to be complete in yaw motion, there are many control algorithms have been proposed. Linear controller design based on linearization around an operating point, such as, pole-placement method [8], [9], [14], linear quadratic regulator (LQR), or optimal control [1], [5], [19]. Another is nonlinear controller design by using Lyapunov-based techniques such as backstepping [7], control Lyapunov function [13], and sliding-mode control (SMC) [10].

In this paper, we apply a new robust nonlinear controller based on concept of *immersion and invariant (I&I)* method, introduced by Astolfi et.al., [3-4] to stabilize the closed-loop system asymptotically around the unstable upright position of the cart body. The proposed controller is designed to cope with exogenous output disturbances and variation of plant parameters. The rest of the paper is outlined as follows. In Section 2, the EL equation of motion and the method of collocated *partial feedback linearization (PFL)* are used for modeling and transform the system into a cascade form. In Section 3, the basic theory of I&I method is presented, namely a set of sufficient conditions for the construction of local (global) stabilizing control laws for general nonlinear affine systems. These results are then used in Section 4 to treat the TWSB robot within four steps: the target system is a priori defined, immersion condition, implicit manifold, and manifold attractivity and trajectory boundedness. Finally, Section VI gives some summarizing remarks and suggestions for future work.

2. Modeling

The TWSB robot is depicted in Fig. 1 consists of a cart body stabilized on two wheels. The mass of each wheel, m is located at the center of the wheel having a radius r. The mass of the cart body, M is assumed to be placed at a distance l from the wheel hub. The mass moment of inertia for the cart body and wheel are denoted as J_c and J_w, respectively. For a stabilization problem, the motion of the robot is restricted in x-y plane. Assuming that the system has two degrees of freedom with the coordinate $q = [\theta\ \phi]^T \in \mathbb{R}^2$, where θ and ϕ are the cart angle (unactuated coordinate) and the wheel angle (actuated coordinate), respectively. A control input is assumed such that motors apply total torque τ on the wheels to navigate the robot a horizontal distance along the xz-plane.

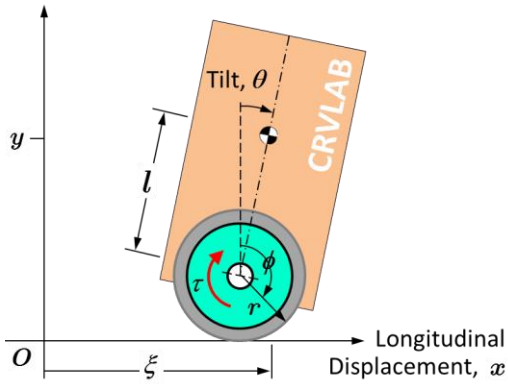

Fig. 1 Configuration of the TWSB robot

2.1 Euler-Lagrange Equation of Motion

The dynamics equation for the TWSB robot is derived from the EL equation, where the Lagrangian function is given by

$$L = \frac{1}{2}\left((2m+M)r^2 + 2J_w\right)\dot{\phi}^2 + \frac{1}{2}(J_c + Ml^2)\dot{\theta}^2 + Mrl\dot{\phi}\dot{\theta}\cos\theta - (2m+M)rg - Mgl\cos\theta \tag{1}$$

and $\frac{d}{dt}\left(\frac{\partial L(\dot{q},q)}{\partial \dot{q}_i}\right) - \frac{\partial L(\dot{q},q)}{\partial q_i} + \frac{\partial D(\dot{q})}{\partial \dot{q}_i} = F(q)u$

with $D(\dot{q}) = \frac{1}{2}c_\xi r^2\dot{\phi}^2 + \frac{1}{2}c_\theta(\dot{\phi}+\dot{\theta})^2$ is Rayleigh' s dissipation function gives equation of motion:

$$\begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix}\ddot{q} + \begin{bmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\tau \tag{2}$$

where

$m_{11} = J_c + Ml^2$, $m_{12} = m_{21} = Mrl\cos\theta$,

$m_{22} = (2m+M)r^2 + 2J_w$,

$h_1 = c_\theta(\dot{\phi}+\dot{\theta}) - Mgl\sin\theta$,

$h_2 = (c_\xi r^2 + c_\theta)\dot{\phi} + c_\theta\dot{\theta} - Mrl\dot{\theta}^2\sin\theta$.

Remark 1: This model combines a friction force occurring on two pairs of contact surfaces: the wheels and the platform, $c_\xi r\dot{\phi}$, and another is a relative motion of the wheel hub and the cart body, $c_\theta(\dot{\phi}+\dot{\theta})^2$.

Remark 2: Due to the lack of control in the first line of (2), it is not possible to fully linearize this underactuated system. However, to obtain a partially linearized underactuated system is possible by using a change of coordinate.

2.2. Collocated Partial Feedback Linearization

A useful technique for control of UMS is the so-called *collocated partial feedback linearization (PFL)* was developed by Spong [18]. By given a change of control and a natural global change of coordinates, it can be shown that the dynamics of the actuated coordinate of any underactuated system can be globally linearized. In this work, we use this procedure to transform (2) into a cascade form, which suitable for selection of a target dynamics in the I&I controller design (Section 4). From the first line of (2), we have

$$\ddot{\theta} = -m_{11}^{-1}m_{12}\ddot{\phi} - m_{11}^{-1}h_1 \tag{3}$$

substituting (3) into the last line of (2), yields

$$(m_{22} - m_{11}^{-1}m_{12}^2)\ddot{\phi} - m_{12}m_{11}^{-1}h_1 + h_2 = \tau \tag{4}$$

Therefore, applying the feedback control law

$$\tau = (m_{22} - m_{11}^{-1}m_{12}^2)u - m_{12}m_{11}^{-1}h_1 + h_2 \tag{5}$$

result in

$$\begin{cases} \ddot{\theta} = -m_{11}^{-1}m_{12}u - m_{11}^{-1}h_1 \\ \ddot{\phi} = u \end{cases} \tag{6}$$

which is valid globally. Let $x = [\theta\ \dot{\theta}\ \dot{\phi}]^T$ be the state, then the state-space equation becomes

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_0(x) + g_0(x)u \\ \dot{x}_3 = u \end{cases} \tag{7}$$

where $g_0(x) \triangleq -m_{11}^{-1}m_{12}$ and $f_0(x) \triangleq -m_{11}^{-1}h_1$.

Remark 3: After the collocated PFL is used, the new control u appears in the dynamics of both nonlinear subsystem (x_1, x_2) and double integrator linear subsystem (x_3).

Remark 4: Olfati-Saber [16] further introduced a global change of coordinates that can decouple these two subsystems but left the linear subsystem invariant.

3. The Basic I&I Method

The present section reviews the basic theoretical results of Astolfi et al. [4], namely a set of sufficient conditions for the construction of globally asymptotically stabilizing state feedback control laws for general nonlinear systems.

Theorem 1: Consider a nonlinear system

$$\dot{x} = f(x) + g(x)u \qquad (8)$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. Let $x^* \in \mathbb{R}^n$ be the equilibrium point to be stabilized and let $p < n$. Suppose we can find mappings

$$\alpha(\cdot): \mathbb{R}^p \to \mathbb{R}^p,\ \pi(\cdot): \mathbb{R}^p \to \mathbb{R}^n,$$
$$c(\cdot): \mathbb{R}^p \to \mathbb{R}^m,\ \phi(\cdot): \mathbb{R}^n \to \mathbb{R}^{n-p},$$
$$\text{and} \quad v(\cdot,\cdot): \mathbb{R}^{n\times(n-p)} \to \mathbb{R}^m$$

such that the following hypothesis hold.

(H1) (Target system) The system

$$\dot{\xi} = \alpha(\xi) \qquad (9)$$

with state $\xi \in \mathbb{R}^p$, has a globally asymptotically stable equilibrium at $\xi^* \in \mathbb{R}^p$ and $x^* = \pi(\xi^*)$

(H2) (Immersion condition) For all $\xi \in \mathbb{R}^p$

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi}\alpha(\xi) \qquad (10)$$

(H3) (Implicit manifold) The following set identity holds

$$M = \{x \in \mathbb{R}^n \mid \phi(x) = 0\}$$
$$= \{x \in \mathbb{R}^n \mid x = \pi(\xi) \ for\ some\ \xi \in \mathbb{R}^p\} \qquad (11)$$

(H4) (Manifold attractivity and trajectory boundedness) All trajectories of the system

$$\dot{z} = \frac{\partial \phi}{\partial x}[f(x) + g(x)v(x,z)] \qquad (12)$$

$$\dot{x} = f(x) + g(x)v(x,z) \qquad (13)$$

are bounded and satisfy

$$\lim_{t\to\infty} z(t) = 0 \qquad (14)$$

Then, x^* is a globally asymptotically stable equilibrium point of the closed-loop system.

$$\dot{x} = f(x) + g(x)v(x, \phi(x)) \qquad \square$$

In this case, we say that the system (8) is *I&I stabilizable* with respect to the target dynamics (9)

Remark 5: The key idea of the Theorem 1 is to achieve the control objective by immersing the plant dynamics into a possible lower-order target system that captures the desired behavior.

4. I&I-Based Controller Design

4.1 Controller Design

We proceed to verify the hypotheses (H1)-(H4) of Theorem 1. Firstly, we choose a target system as a simple second-order pendulum.

(H1) (Target system) We defined the target dynamics as

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2 \end{cases} \qquad (15)$$

where $\xi \in \mathbb{R}^2$, $V: \mathbb{R} \to \mathbb{R}$ is the potential energy of the system and $R: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a damping function which are to be chosen. The target system (11) has a stable equilibrium at ξ^* with energy function $H(\xi_1, \xi_2) = \frac{1}{2}\xi_2^2 + V(\xi_1)$.

To ensure the stability at the equilibrium we introduce the following assumption.

Assumption 1:

(i) The potential energy function $V(\xi_1)$, satisfies

$$V'(\xi_1^*) = 0 \quad \text{and} \quad V''(\xi_1^*) > 0,$$

(ii) The damping matrix is $R(\xi^*) \geq 0$. □

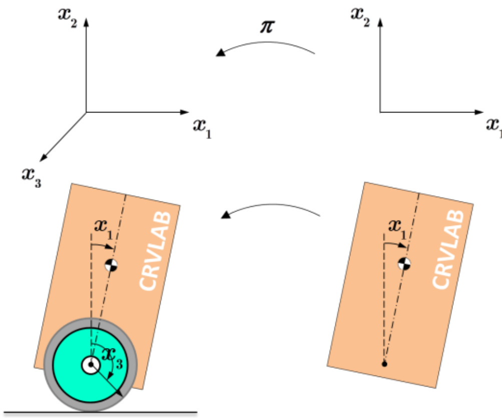

Fig. 2 TWSB robot and target dynamics

(H2) (Immersion condition). Given the control objectives and our choice of target dynamics is as depicted in Fig. 2. A natural selection of the mapping $\pi(\cdot)$ is

$$\pi(\xi) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi_1, \xi_2) \end{bmatrix} \tag{16}$$

where $\pi_3(\cdot)$ is a function to be defined. With this choice of $\pi(\cdot)$ and the target dynamics the PDE (10) reduces to

$$\begin{bmatrix} \xi_2 \\ f_0(\xi_1, \xi_2) + g_0(\xi_1) c(\pi(\xi)) \\ c(\pi(\xi)) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \dfrac{\partial \pi_3(\xi)}{\partial \xi_1} & \dfrac{\partial \pi_3(\xi)}{\partial \xi_2} \end{bmatrix} \begin{bmatrix} \xi_2 \\ -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2 \end{bmatrix} \tag{17}$$

Next we choose $\pi_3(\xi)$ and $c(\pi(\xi))$ to satisfy the above equation as follows: the first row of (17) is already satisfied. Consider the last two rows

$$f_0(\xi_1, \xi_2) + g_0(\xi_1) c(\pi(\xi)) = -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2 \tag{18}$$

$$c(\pi) = \frac{\partial \pi_3}{\partial \xi_1} \xi_2 - \frac{\partial \pi_3}{\partial \xi_2} \left(V'(\xi_1) + R(\xi_1, \xi_2)\xi_2 \right) \tag{19}$$

Replacing $c(\cdot)$ from (19) in (18) yields the PDE

$$-\left(g_0(\xi_1) \frac{\partial \pi_3}{\partial \xi_1} + \Delta(\xi_1) R(\xi_1, \xi_2) \right) \xi_2 = f_0(\xi_1, \xi_2) + \Delta(\xi_1) V'(\xi_1) \tag{20}$$

Defining

$$V'(\xi_1) \triangleq -\frac{f_0(\xi_1, \xi_2)}{\Delta(\xi_1)}, \quad R(\xi_1, \xi_2) \triangleq -\frac{g_0(\xi_1)}{\Delta(\xi_1)} \frac{\partial \pi_3}{\partial \xi_1}$$

with $\Delta(\xi_1) \triangleq 1 - g_0(\xi_1) \dfrac{\partial \pi_3}{\partial \xi_2}$ and $\Delta(0) < 0$.

Clearly, (20) holds by Assumption 1, hence (H2) holds.

(H3) (Implicit manifold) The implicit manifold can be described by $\phi(x) = x_3 - \pi_3(x_1, x_2) = 0$ and the off-the-manifold coordinate is

$$z = x_3 - \pi_3(x_1, x_2) \tag{21}$$

(H4) (Manifold attractivity and trajectory boundedness) The off-the-manifold dynamics are

$$\dot{z} = -\frac{\partial \pi_3}{\partial x_1} x_2 - \frac{\partial \pi_3}{\partial x_2} f_0(x_1, x_2) + \Delta(x_1) v(x, z)$$

Selecting

$$v(x, z) = \frac{1}{\Delta(x_1)} \left(-\gamma z + \frac{\partial \pi_3}{\partial x_1} x_2 + f_0(x_1, x_2) \frac{\partial \pi_3}{\partial x_2} \right) \tag{22}$$

which is well-defined locally around the origin by assumption, yields $\dot{z} = -\gamma z$. Finally, the I&I control law is obtained by replacing $z = \phi(x)$ in (22).

4.2 Stability Result

We establish boundedness of the trajectories of the closed-loop system (8) with the control law (22) and the off-the-manifold dynamics $\dot{z} = -\gamma z$. In other worlds, consider the trajectories of the system (12), (13), namely

$$\begin{cases} \dot{z} = -\gamma z \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = f_0(x_1, x_2) + g_0(x_1) v(x, z) \\ \dot{x}_3 = v(x, z) \end{cases} \quad (23)$$

with $v(\cdot)$ given by (22), are bounded. Towards this end, note that from the physical properties of the TWSB robot, there exist $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $m_0 > 0$ such that, locally around zero, $\Delta(x_1) \le -\varepsilon_1$, $R(x_1, x_2) \ge \varepsilon_2$ and $m_0 > | g_0(x_1) |$. Note now that, using the functions $V'(\cdot)$ and $R(\cdot)$ defined in the foregoing equation, the first three equations of (23) can be rewritten in the form

$$\begin{cases} \dot{z} = -\gamma z \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = -V'(x_1) - R(x_1, x_2) x_2 - \gamma \dfrac{g_0(x_1)}{\Delta(x_1)} z \end{cases} \quad (24)$$

Consider the function $H(x_1, x_2) = \frac{1}{2} x_2^2 + V(x_1) + \frac{\gamma}{2\varepsilon_2 \varepsilon_1^2} z^2$ and note that, $V''(0) = 0$ and $V''(0) = -\frac{f_0'(0,0)}{\Delta(0)} > 0$. Hence, the function $H(x_1, x_2)$ is positive definite with a local minimum at the origin. Differentiating along the trajectories of (24) yields

$$\begin{aligned} \dot{H}(x_1, x_2) &= -R(x_1, x_2) x_2^2 - \gamma \frac{g_0(x_1)}{\Delta(x_1)} x_2 z - \frac{\gamma^2}{2\varepsilon_2 \varepsilon_1^2} z^2 \\ &\le -\varepsilon_2 x_2^2 - \gamma \frac{g_0(x_1)}{\Delta(x_1)} x_2 z - \frac{\gamma^2}{2\varepsilon_2 \varepsilon_1^2} z^2 \\ &\le -\frac{\varepsilon_2}{2} x_2^2 - \frac{\gamma^2 (1 - m_0^2)}{2\varepsilon_2 \varepsilon_1^2} z^2 \end{aligned}$$

where we have used Young's inequality:

$$\gamma \left| \frac{g_0(x_1)}{\Delta(x_1)} \right| x_2 z \le \gamma \frac{m_0}{\varepsilon_1} | x_2 || z | \le \frac{\varepsilon_2}{2} x_2^2 + \frac{\gamma^2 m_0^2}{2\varepsilon_2 \varepsilon_1^2} z^2$$

From (15) and (16), we can obtain that states are bounded and converge to the origin. We know that the off-the-manifold coordinate z is bounded and $\lim_{t \to \infty} z(t) = 0$. Finally, boundedness of x_3 follows from the fact that $x_3(t) = z(t) + \pi_3(x_1(t), x_2(t)) - z(0) + x_3(0)$ and the right-hand side is bounded. The above discussion on the control synthesis can be summarized in the following proposition which is the main result of this paper.

Proposition 1: The closed-loop system (7) with the control law (21)-(22) is asymptotically stable at the origin. □

Proof: From the derivations above it is clear that Proposition 1 can be easily proved, but omitted here for brevity. □

5. Simulation Results

The I&I-based controller for stabilization problem of the TWSB robot has been designed in the Section 4. The hypothesis (H1)-(H4) were given the I&I control law (21)-(22), whereas some types of choices for the off-manifold variable $\pi_3(\cdot)$ are designed as in Table 1.

Table. 1 Design type of $\pi_3(\cdot)$

Type of $\pi_3(x_1, x_2)$	$\partial \pi_3(x_1, x_2) / \partial x_1$	$\partial \pi_3(x_1, x_2) / \partial x_2$
$-k_1 x_1 - k_2 x_2$	$-k_1$	$-k_2$
$-k_1 x_1 - \dfrac{k_2}{\cos x_1} x_2$	$-k_1 - k_2 (\sec x_1 \tan x_1) x_2$	$-\dfrac{k_2}{\cos x_1}$
$-\left(\dfrac{1}{\cos x_1} + \dfrac{1}{k_1} \cos^2 x_1 \right) x_2 + \psi(x_1)$, with $\psi(x_1) = k_1 x_1$	$-\left(\sec x_1 \tan x_1 - \dfrac{\sin 2x_1}{k_1} \right) x_2 - k_2$	$-\left(\dfrac{1}{\cos x_1} + \dfrac{\cos^2 x_1}{k_1} \right)$

For testing performance of the proposed controller, the model (7) is used as the nominal model of the TWSB robot, where $m = 0.22$ kg, $M = 1.11$ kg, $J_c = 3.47 \times 10^{-4}$ kg-m², $J_w = 4.63 \times 10^{-4}$ kg-m², $r = 0.055$ m, $l = 0.025$ m, $c_\theta = 0.01$, and $c_\xi = 0.1$. The simulation results for each design of $\pi_3(\cdot)$ show in Fig. 3 and Fig. 4 for the nominal model case and the perturbed model case, respectively. In both cases, the responses in the red line represent for the first two type of $\pi_3(\cdot)$ correspond to a set of controller gains $k_1 = 25$, $k_2 = 3$, $\gamma = 10$, and they are identically. The blue line represents for the last type of $\pi_3(\cdot)$ correspond to a set of controller gains $k_1 = 3,\ k_2 = 7, \gamma = 7$.

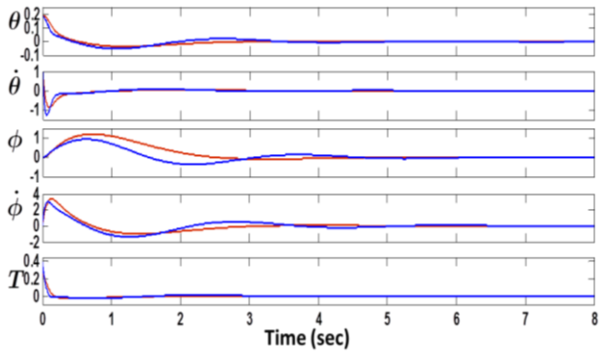

Fig. 3 Time response of the nominal case of the states and the control input

The robustness of the I&I controller is demonstrated by adding 10% variation of the nominal parameters m_{ij} and h_i , $\forall i, j = 1, 2$ into the nominal model (7). Moreover, we assume that there are exogenous disturbances perturbing on the system outputs by $\theta(t) = \theta_{\text{nominal}}(t) + 0.01\sin(200\pi t)$ rad, and $\phi(t) = \phi_{\text{nominal}}(t) + 0.01\sin(200\pi t)$ rad. The simulation result for this case show in Fig. 4, where the controller gains are given same as in the previous simulation.

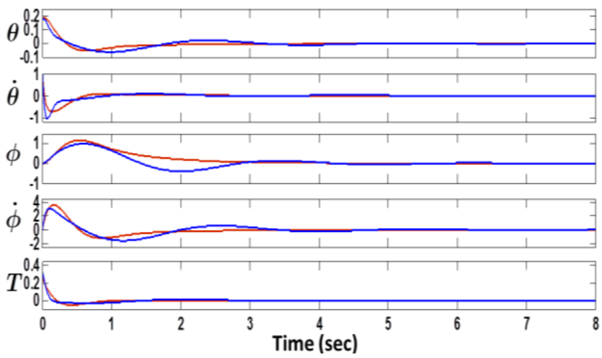

Fig. 4 Time response of the perturbed case of the states and the control input

In the Figure 3 and Figure 4 show that the I&I-based controller is performed well in both two case, i.e., nominal model case and perturbed model case. Due to a design of damping function $R(\cdot)$ that be a positive definite, the proposed I&I controller then has inherent property of robust stability. Therefore, the ability of stabilization and disturbance rejection for each type of $\pi_3(\cdot)$ in the controller design is quite similar. The TWSB robot can be stabilized at the upright position while wheels position is brought to the initial state.

6. Concluding Remarks

In this paper, a nonlinear controller design, for stabilizing the TWSB robot under some disturbances, has been proposed using I&I technique. The modeling of the TWSB robot is based on the EL equation of motion and using the PFL to transform the state-space robot model into a feedback form suitable for designing of the I&I controller. The procedures of the controller design follow from Astolfi et al. [4], based on selection of target system and a certain selection of a stable potential energy function and positive damping function. The stability proof and the simulation results have demonstrated that all trajectories of the closed-loop systems are asymptotically stable. Therefore, the TWSB robot can be stabilized at the upright position while wheels position is brought to the initial state.

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